Fantastic LI – Ideals and Vague Fantastic LI- Ideals of Lattice Implication Algebras

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Abstract: The authors first introduce the notion of fantastic LI –ideals of lattice implication algebra. We prove that every fantastic LI –ideal of a lattice implication algebra L is a LI-ideal of L and converse need not to be true. Then we give a condition LI- ideal should be a fantastic LI –ideal. Also we prove that ILI – ideals of lattice implication algebra are fantastic LI –ideals and fantastic LI –ideal need not to be ILI – ideal. Finally we give the extension property for fantastic LI –ideals. Later we apply the vague concept to the notion of fantastic LI-ideals and discussing relation among vague fantastic LI-ideals, vague LI- ideals and Vague Implicative LI – ideals and properties of vague fantastic LI-ideals.

Keywords: Lattice Implication algebra, fantastic LI –ideals, vague set, vague fantastic LI - ideals.

1. Introduction


The concept of fuzzy set was introduced by Zadesh [5] to deal with vagueness. Gau W. L., Buehrer D. J. [2] propounded the postulation of vague set in 1993. We [T. Anitha, V. Amarendra Babu] [7,8,9] apply the concept of vague to various LI-ideals and introduced the concepts of Vague LI – ideals ,Vague implicative LI- ideals, vague positive implicative LI- ideals etc. on lattice implication algebras.

The main aim of this paper is to introduce the notion of fantastic LI –ideals of lattice implication algebra. Then discuss the relations among the fantastic LI –ideals, LI-ideals and ILI –ideals of lattice implication algebra. We give the extension property for fantastic LI –ideals. Later we apply the vague concept to the notion fantastic LI –ideals and discussing properties of vague fantastic LI - ideals.
2. Preliminaries

**Definition 2.1**[13]: The complemented bounded lattice \((\mathcal{L}, \lor, \land, \neg, 0, I)\) is called a lattice implication algebra, if the following axioms hold, \(\forall\  p, q, r \in \mathcal{L},\)

1. \(p \rightarrow (q \rightarrow r) = q \rightarrow (p \rightarrow r);\)
2. \(p \rightarrow p = I;\)
3. \(p \rightarrow q = q' \rightarrow p';\)
4. \(p \rightarrow q = q \rightarrow p = I \) implies \(p = q;\)
5. \((p \rightarrow q) \rightarrow q = (q \rightarrow p) \rightarrow p;\)
6. \((p \lor q) \rightarrow r = (p \rightarrow r) \land (q \rightarrow r);\)
7. \((p \land q) \rightarrow r = (p \rightarrow r) \lor (q \rightarrow r).\)

Throughout this paper \(\mathcal{L}\) represents the Lattice implication algebra.

**Theorem 2.2**[13]: Let \(p, q, r \in \mathcal{L}\) then,

1. \(I \rightarrow p = p, p \rightarrow 0 = p', 0 \rightarrow p = I \) and \(p \rightarrow I = I.\)
2. \(p \leq q \) if and only if \(p \rightarrow q = I.\)
3. \(p \rightarrow q \leq (q \rightarrow r) \rightarrow (p \rightarrow r).\)
4. \(p \rightarrow q \leq (r \rightarrow p) \rightarrow (r \rightarrow q).\)
5. \(p \leq (p \rightarrow q) \rightarrow q.\)

**Definition 2.3**[14]: A \(\mathcal{L}\) is lattice H implication algebra if \(p \lor q \lor ((p \land q) \rightarrow r) = I\) for all \(p, q, r \in \mathcal{L}\).

**Definition 2.4**[9]: A nonempty subset \(A\) of \(\mathcal{L}\) is an LI - ideal of \(\mathcal{L}\) if:

1. \(0 \in A;\)
2. \(\forall\  p, q \in \mathcal{L}, (pq)' \in A, q \in A \) imply \(p \in A.\)

**Definition 2.5**[12]: A nonempty subset \(A\) of \(\mathcal{L}\) is an ILI - ideal of \(\mathcal{L}\) if:

1. \(0 \in A;\)
2. \(\forall\  p, q, r \in \mathcal{L}, ((pq)'qr)' \in A, r \in A \) imply \((pq)' \in A.\)
Definition 2.6[11]: A nonempty subset F of ℳ is a fantastic filter of ℳ if:

(1) I ∈ F;

(2) ∀ p, q ∈ ℳ, r(pq) ∈ F and p ∈ F imply ((pq)q)p ∈ F.

Theorem 2.7 [1, theorem 2.2]: Let I be a LI – ideal of ℳ and p ∈ I. If q ≤ p then q ∈ 1 for all y ∈ ℳ.

Corollary 2.8 [12, Corollary 3.15]: Let I be a LI – ideal of ℳ. Then I is a ILI -ideal of ℳ if and only if (p(qp)′) ∈ I implies p ∈ I for all p, q, ∈ ℳ.

Theorem 2.9[12, theorem 2.5]: Let I be a nonempty subset of ℳ. Then I is a LI -ideal of ℳ if and only if for all p, q ∈ I and r ∈ ℳ, (rp)′ ≤ q implies r ∈ I.

Definition 2.10[2]: A vague set H in the universal of discourse U is characterised by two membership functions given by truth membership function tH and false membership function fH. Where tH(p) is a lower bound of the grade of membership of p derived from the “evidence for p”, and fH(p) is a lower bound on the negation of p derived from the “evidence against p” and tH(p) + fH(p) ≤ 1. Thus the grade of membership of p in the vague set A is bounded by subinterval [tH(p), 1 – fH(p)] of [0, 1]. The vague set H is written as H = { (p, [tH(p), fH(p)]) / p ∈ P}. Where the interval [tH(p), 1 – fH(p)] is called the value of p in the vague set H and denoted by VtH(p).

Definition 2.11[2]: Let A be a vague set of a universe U with the truth membership function tH and the false membership function fH. For any α, β ∈ [0, 1] with α ≤ β, the (α, β) – cut or vague cut of a vague set H is a crisp subset H (α, β) of the set P given by H (α, β) = { p ∈ U / VtH(p) ≥ [α, β] }.

Definition 2.12 [2]: The α – cut, Hα of the vague set H is the (α, α) – cut of H and hence given by Hα = {p ∈ U / tH(p) ≥ α}.

Definition 2.13[6]: A vague set A of ℳ is VLI - ideal (briefly VLI – ideal) of L if:

(1) ∀ p ∈ ℳ, VΑ(0) ≥ VΑ(p),

(2) ∀ p, q ∈ ℳ, VΑ(p) ≥ imin{VΑ((pq)′) , VΑ(q)}.

Definition 2.14[8]: A vague set A of ℳ is VILI – ideal (briefly VILI – ideal) of L if:

(1) ∀ p ∈ ℳ, VΑ(0) ≥ VΑ(p),

(2) ∀ p, q, r ∈ ℳ, VΑ((pq)′) ≥ imin{VΑ(((pq)′q)′r)′) , VΑ(r)}.

Theorem 2.15[7, theorem 26]: Let A be a VLI-ideal of ℳ. Then the following are equivalent:

(1) A is a VILI- ideal of ℳ.

(2) VΑ(p) ≥ VΑ((pqp)′)′.
3. Fantastic LI – ideals

**Definition 3.1:** A nonempty subset $I$ of $\mathcal{L}$ is said to be a fantastic LI – ideal (shortly FLI) of $\mathcal{L}$ if it satisfies the following conditions:

1. $0 \in I$,
2. $\forall p, q, r \in I$, $((pr)q) \in I$ and $q \in I$ imply $((rp)r)q \in I$.  

Clearly the subset $\{0\}$ of $\mathcal{L}$ is a FLI of $\mathcal{L}$.

**Example 3.2:** Let $L = \{0, p, q, r, s, I\}$.

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Define $\lor$, $\land$ and $\rightarrow$ operations on $L$ as follows:

$p' = p0$, $p \lor q = (pq)q$, $p \land q = ((pq')q')$ for all $p, q \in \mathcal{L}$. Then $(\mathcal{L}, \lor, \land, \rightarrow, \lor, 0, 1)$ is a lattice implication algebra.

Clearly the subset $A = \{0, r\}$ of $\mathcal{L}$ is a FLI ideal of $\mathcal{L}$.

**Example 3.3:** Let $\mathcal{L} = \{1, 3, 5, 15, 25, 75\}$.

Define $\lor$, $\land$, $\lor$ and $\rightarrow$ on the set $L$ as follows:

Let $p, q \in L$ then $p \lor q = \text{LCM} \{p, q\}$, $p \land q = \text{GCD} \{p, q\}$, $p' = 75/p$ and $p \rightarrow q = p' \lor q$.
Then the set \{ \land, \lor, \land', \lor', \rightarrow, 1, 75 \} is a lattice implication algebra.

Clearly the subsets \( A = \{ 1, 15 \} \) and \( B = \{ 1, 3, 25 \} \) of \( L \) are FLI of \( L \).

**Theorem 3.4:** Every FLI of \( L \) is a LI – ideals of \( L \).

**Proof:** Let \( I \) be a FLI of \( L \). Obviously \( 0 \in I \).

Let \( q \in I \) and \( (pq)' \in I \).

Then \( (pq)' = ((0p)' q)' \in I \) and \( q \in I \).

By (3.1), we get \(((0p)p)' \in I \).

Clearly \( (((0p)p) 0)' = (((1p)0)' = (p)'' = p \in I \).

Hence \( I \) is a LI – ideals of \( L \).
**Theorem 3.5:** An LI – ideal of L is a FLI of L if and only if it satisfies:

\[(pr)' \in I \text{ implies } (((rp)p)r)' \in I \text{ for all } p, q, r \in L\]  

**Proof:** Suppose that I is a FLI of L and p, r \in I such that (pr)' \in I.

Then (((pr)'0)' = (pr)' \in I and 0 \in I.

By (3.1), we get (((rp)p)r)' \in I.

Conversely suppose that I is a LI – ideal of L and satisfying (3.2).

Let p, q, r \in I be such that (pr)' q)' \in I and q \in I.

Then (pr)' \in I since I is a LI – ideal and by 3.2, we get (((rp)p)r)' \in I.

So I is a FLI of L.

**Theorem 3.6:** Every ILI - ideal of L is FLI of L.

**Proof:** Let I be an ILI – ideal of L and (pr)' \in I for p, q \in I.

Clearly r \leq (((rp)p)r \Rightarrow rp \geq (((rp)p)r)p

\Rightarrow (rp)' \leq (((rp)p)r)p'

\Rightarrow (rp)' \leq (p'( ((rp)p)r ))'.

Taking (((rp)p)r )' = t, we get (rp)' \leq (p't)'.

Clearly t(p t)' \geq t(rp)' = (((rp)p)r ) (rp)' = (rp) (((rp)p)r ) = (((rp)p) ((rp)r) = pr.

So (t(p t)' )' \leq (pr)' and (pr)' \in I implies (t(p t)' )' \in I. (By theorem 2.7)

By corollary 2.2, we get t = (((rp)p)r )' \in I.

By (3.2), we get I is a FLI of L.

The converse of the theorem 3.6 is needed not to be true as seen in the following example.

**Example 3.7:** From the example 3.2, the subset A = \{0, r\} of L is a FLI of L. But it is not a ILI – ideal of L because (((p \rightarrow q) \rightarrow q)' \rightarrow 0)' = 0 \in A and 0 \in A, but (p \rightarrow q)' = s \notin A.

**Lemma 3.8:** In lattice H implication algebra L, every FLI is an ILI- ideal.

**Proof:** Let I be a FLI- ideal of lattice H implication algebra L.

From theorem 3.4 and theorem 3.3[12], we get I is a ILI- ideal of L.

**Example 3.9:** Clearly the FLI -ideals A = \{1, 15\} and B = \{1, 3, 25\} of L in the example 3.3 are ILI - ideals of L.
Theorem 3.10: Let I be a nonempty subset of L. Then I is a FLI of L if and only if I' is a fantastic filter of L.

Proof: Suppose that I is a FLI of L.

Let p, q, r ∈ I be such that r(qp), r ∈ I'.

Then (r(qp))', r' ∈ I ⇒ ((qp)' r')', r' ∈ I

⇒ (((pq)q)p)' ∈ I (By (3.1))

⇒ (((pq)q)p) ∈ I'

So I' is a fantastic filter of L.

Conversely suppose that I' is a fantastic filter of L.

Let p, q, r ∈ I be such that ((pr)' q)' , q ∈ I.

Implies ((pr)' q) , q' ∈ I' ⇒ q' (pr), q' ∈ I'

⇒ ((rp)p)r ∈ I' (by definition 2.6)

⇒ (((rp)p)r)' ∈ I.

Hence I is a FLI-ideal of L.

Theorem 3.11: Let I1 and I2 be LI-ideals of L such that I1 ⊆ I2. If I1 is FLI-ideal then so is I2.

Proof: Let I1 be a FLI-ideal of L and I2 be a LI-ideal of L such that I1 ⊆ I2.

Let p, q ∈ L be such that (pq)' ∈ I2. It is sufficient to show that (p((pq)q))' ∈ I2.

Clearly (p((pq)q))' = (pq)(pq)' = 0 ∈ I1.

Taking (pq)q = t and by 3.2, we get (((tp)t) t)' ∈ I1 ⊆ I2.

That is (((pq)q)p)((pq)q) y ∈ I2 ⇒ ((pq)((pq)q)p) q)' ∈ I2

⇒ (((pq)q)p) q)' ∈ I2

Since I2 is LI-ideal of L, we have (((pq)q)p) q)' ∈ I2.

Consider (((qp)p)q)'((pq)q)p) q) = (((pq)q)p) q) ((pq)pq)

≥ ((qp)p) (((pq)q)p)

= (((pq)q)p) (qp)

= q((pq)q)

= (pq)(qq)
Then (((pq)p)q)'((((pq)q)p)p) q') ≥ I implies (((pq)p)q)'((((pq)q)p)p) q')' ≤ 0 ∈ I_2.

By 2.3, we get (((pq)p)q)' ∈ I_2.

By 3.2, we get I_2 is a FLI of L.

4. Vague fantastic LI - ideal

**Definition 4.1:** A vague set H in L is said to be a vague fantastic LI - ideal (shortly VFLI- ideal) of L if it satisfies the following conditions:

1. \( \forall p \in L, V_H(0) ≥ V_H(p) \),
2. \( \forall p, q, r \in L, V_H(((rp)p)r) ≥ \text{imin}\{V_H((pr)q'), V_H(q)\} \). …………………..(4.1)

**Example 4.2:** Let L be lattice implication algebra in the example 3.2. Then the vague set G defined by

\[
G = \{0, [0.7, 0.2]), (p, [0.7, 0.2]), (q, [0.5, 0.3]), (r, [0.7, 0.3]), (s, [0.5, 0.3]), (l, [0.5, 0.3])\}
\]

is a VFLI – ideal of L.

**Example 4.3:** Let L be lattice H implication algebra in the example 3.3. Then the vague set H defined by

\[
H = \{1, [0.7, 0.2]), (3, [0.7, 0.2]), (5, [0.5, 0.3]), (15, [0.5, 0.3]), (25, [0.7, 0.2]), (75, [0.5, 0.3])\}
\]

is a VFLI – ideal of L.

**Theorem 4.4:** Every VFLI- ideal of L is a VLI – ideal of L.

**Proof:** Let H be a VFLI- ideal of L.

Taking \( r = 0 \) in the equation( 4.1), we get

\[
V_H(((0p)p)0)' ≥ \text{imin}\{V_H(((0)q)') , V_H(q)\}
\]

\[
V_H(((lp)0)' ≥ \text{imin}\{V_H((p')q') , V_H(q)\}
\]

\[
V_H((p0)' ≥ \text{imin}\{V_H((pq)' , V_H(q)\}
\]

\[
V_H(p) ≥ \text{imin}\{V_H((pq)' , V_H(q)\}.
\]

So H is a VLI- ideal of L.

**Theorem4.5:** A VLI- ideal of L is a VFLI- ideal of L if and only if it satisfies the condition:

\[
V_H((pr)') ≤ V_H(((rp)p)r)' \), for all p, q, r ∈ L………………….. (4.2)
Proof: Suppose that H is a VFLI- ideal of L.
Taking q = 0 in the equation (4.1), we get
\[ V_H(((rp)p)r') \geq \text{imin}\{V_H(((pr)'0') , V_H(0)) \}
\geq \text{imin}\{V_H((pr') , V_H(0)) \}
= V_H((pr')). \]

Conversely suppose that H is a VLI- ideal of L and having the inequality (4.2).
Then we have
\[ V_H(((rp)p)r') \geq V_H((pr') \geq \text{imin}\{V_H(((pr)'q') , V_H(q)\}. \]
So H is a VFLI- ideal of L.

Theorem 4.6: Every VILI-ideal is a VFLI of L.

Proof: Let H be VILI – ideal of L.
Clearly \( r \leq ((rp)p)r \Rightarrow rp \geq (((rp)p)r)p \)
\[ \Rightarrow (rp)' \leq (((rp)p)r )p)' \]
\[ \Rightarrow (rp)' \leq \{p' (((rp)p)r )'\) for all for p, q and r \in I. \]

Taking \(((rp)p)r )' = t, we get \((rp)' \leq (p't)'. \)

Clearly \( t(p't)' \geq t(rp)' = (((rp)p)r )' (rp)' = (rp) (((rp)p)r ) = ((rp)p) ((rp)r) = pr. \)
So \( tt(p't)' \leq (pr)' \) and H is a order reversing, we have \( V_H ((pr') \leq V_H ((tt(p't)' )'). \)

By theorem 2.15, we get \( V_H(((rp)p)r') = V_H(t) \geq V_H ((tt(p't)' )') \geq V_H ((pr')). \)

Hence by (4.2), H is a VFLI – ideal of L.

The converse of the theorem 3.6 is needed not to be true as seen in the following example.

Example 4.7: From the example 4.2, the vague set
\[ G = \{0, [0.7, 0.2]) , (p, [0.7, 0.2]), (q, [0.5, q]), (r, [0.7, 0.3]), (s, [0.5, 0.3]), (t, [0.5, 0.3])\} \]
of L is a VFLI ideal of L. But it is not a VILI – ideal of L because
\[ V_G (((p \rightarrow q)' \rightarrow q)' \rightarrow 0') = V_G (0) = [0.7, 0.2] , V_G (0) = [0.7, 0.2] \text{ and } V_G(p \rightarrow q)' = s = [0.5, 0.3]. \]

Clearly \( V_G(p \rightarrow q)' < \text{imin}\{V_G (((p \rightarrow q)' \rightarrow q)' \rightarrow 0')\), V_G (0)\} \).

Lemma 4.8: In lattice H implication algebra L, every VFLI- ideal is a VILI- ideal of L.
**Proof:** Let $H$ be VFLI-ideal of lattice $H$ implication algebra $L$.

From theorem 4.4 and theorem 3.5[12], we get $H$ is a VILI-ideal of $L$.

**Example 4.9:** Clearly the VFLI-ideal

$$H = \{ (1, [0.7, 0.2]), (3, [0.7, 0.2]), (5, [0.5, 0.3]), (15, [0.5, 0.3]), (25, [0.7, 0.2]), (75, [0.5, 0.3]) \}$$

of $L$ in the example 4.3 is VILI-ideals of $L$.

**Theorem 4.10:** Let $H$ be a vague set of $L$. Then $A$ is a VFLI – ideal of $L$ if and only if $H(\alpha, \beta)$ is an FLI – ideal when $H(\alpha, \beta) \neq \emptyset, \alpha, \beta \in [0, 1]$.

**Proof:** Suppose that $H$ is a VFLI – ideal of $L$ and $\alpha, \beta \in [0, 1]$.

If $H(\alpha, \beta) \neq \emptyset$ then there exist $p \in L$ such that $V_H(p) \geq [\alpha, \beta]$.

Clearly $V_H(0) \geq V_H(p) \geq [\alpha, \beta]$. So $0 \in H(\alpha, \beta)$.

Let $p, q, r \in L$ such that $((pr)q)' \in H(\alpha, \beta)$ and $q \in H(\alpha, \beta)$.

Then $V_H(((pr)q)') \geq [\alpha, \beta]$ and $V_H(q) \geq [\alpha, \beta]$.

By (4.1), we get $V_H(((pr)q)) \geq \text{imin} \{V_H(((pr)q)'), V_H(q)\} \geq [\alpha, \beta]$.

So $((qp)p)r)' \in H(\alpha, \beta)$.

Consequently, $H(\alpha, \beta)$ is an FLI-ideal of $L$.

Conversely, Suppose that $H(\alpha, \beta) \neq \emptyset$ is a FLI – ideal of $L$ where $\alpha, \beta \in [0, 1]$.

For all $p \in L$, $H_H(p) \neq \emptyset$ since $p \in H_H(p)$, and $H_H(p)$ is an FLI ideal of $L$.

So $0 \in H_H(p)$, then we have $V_H(0) \geq V_H(p)$.

Let $p, q, r \in L$, let us consider $[\alpha, \beta] = \text{imin} \{V_H(((pr)q)'), V_H(q)\}$.

Then $((pr)q)' \in H(\alpha, \beta)$, this means that $H(\alpha, \beta) \neq \emptyset$ and then $H(\alpha, \beta)$ is an IIL – ideal of $L$.

By (3.1), we have $(((rp)p)r)' \in H(\alpha, \beta)$. So the following inequality hold

$$\forall x, y, z \in L, V_A (((rp)p)r)' \geq [\alpha, \beta] = \text{imin} \{V_H(((pr)q)'), V_H(q)\}.$$ 

Thus $H$ is a VF LI – ideal of $L$.

**Corollary 4.11:** Let $H$ be a vague set of $L$. Then $H$ is a VFLI – ideal of $L$ if and only if $H_\alpha$ is an FLI – ideal when $H_\alpha \neq \emptyset, \alpha \in [0, 1]$.

**Theorem 4.12:** Let $J$ and $K$ be VFLI-ideals of $L$ such that $J \subseteq K$. If $J$ is a VFLI-ideal of $L$, then so is $K$.

**Proof:** Let $J$ and $K$ are VFLI-ideals of $L$ such that $J \subseteq K$. 


Since \( J \subseteq K \), that is \( V_J(p) \leq V_K(p) \) \( \forall p \in L \), clearly we have \( J_\alpha \subseteq K_\alpha \) for every \( \alpha \in [0, 1] \).

Since \( J \) is a VFLI-ideal of \( L \) then \( J_\alpha \neq \emptyset \) is FLI – ideal where \( \alpha \in [0, 1] \).

By theorem 3.11, \( K_\alpha \neq \emptyset \) is a FLI – ideal.

By corollary 4.11, \( K \) is a VFLI – ideal of \( L \).

**Theorem 4.13:** Let \( I \) be an FLI – ideal of \( L \). The vague set \( H \) defined by

\[
V_H(p) = [\alpha, \alpha] \text{ if } p \in I \\
= [0, 0] \text{ if } p \notin I
\]

is a VFLI – ideal of \( L \), where \( \alpha \in [0, 1] \).

**Proof:** Let \( H \) be a vague set of \( L \), defined by

\[
V_H(p) = [\alpha, \alpha] \text{ if } p \in I \\
= [0, 0] \text{ if } p \notin I, \text{ where } \alpha \in [0, 1].
\]

Since \( I \) is a FLI – ideal of \( L \), we have \( 0 \in I \) and so \( V_H(0) = [\alpha, \alpha] \geq V_H(p) \) \( \forall p \in L \).

Let \( p, q \in L \) and (((rp)p)r)' \( \in I \) then

\[
V_A (((rp)p)r)' = [\alpha, \alpha] \geq \text{imin} \{ V_H(((pr)'q)'), V_H(q) \}.
\]

Suppose (((rp)p)r)' \notin I \) then either ((pr)'q)' \notin I or q \notin I and so

\[
V_A (((rp)p)r)' = [0.0] = \text{imin} \{ V_H(((pr)'q)'), V_H(q) \}.
\]

Hence \( H \) is a VFLI – ideal of \( L \).

**Theorem 4.14:** Let \( H \) be a VFLI – ideal of \( L \). Then \( I_H = \{ p \in L / V_H(p) = V_H(0) \} \) is an FLI – ideal of \( L \).

**Proof:** Since \( I_H = \{ p \in L / V_H(x) = V_H(0) \} \), obviously \( 0 \in I_H \).

Let \( p, q, r \in L \) and ((pr)'q)' \( \in I_H \), then \( V_H(((pr)'q')) = V_H(q) = V_A (0) \).

Since \( H \) is a VFLI – ideal, then we have \( V_A (((rp)p)r)' \geq \text{imin} \{ V_H(((pr)'q)'), V_H(q) \} \)

\[
= V_H(0).
\]

And \( V_H(0) \geq V_A (((rp)p)r)' \), then \( V_A (((rp)p)r)' = V_A (0) \). Thus \( ((rp)p)r)' \in I_H \).

It follows that \( I_H \) is an FLI – ideal of \( L \).

**Example 4.15:** Let \( L \) be the lattice implication algebra in the example 3.3. Then the vague set \( H \) defined by

\[
H = \{ (1, [0.7, 0.2]), (3, [0.7, 0.2]), (5, [0.5, 0.3]), (15, [0.5, 0.3]), (25, [0.7, 0.2]), (75, [0.5, 0.3]) \}
\]
is VFLI – ideal of L.
Clearly $I_{VH} = \{p \in L \mid V_{H}(x) = V_{H}(0) \} = \{1,3,25\}$ is a FLI – ideal of L.

References


[14] Y. Xu and K.Y. Qin, Lattice H implication algebras and lattice implication algebras classes, J. Hebel Mining and Civil Engineering Institute, 3,(1992), 139-143.