FREE CONVECTION BOUNDARY LAYER FLOW PAST A INCLINED FLAT PLATE EMBEDDED IN A POROUS MEDIUM FILLED WITH A NANOFLUID

G. Bal Reddy[1], B. Shankar Goud[2] and MN. Raja Shekar[3]

[1]Department of Mathematics, Mahatma Gandhi Institute of Technology, Gandipet Main Road, Kokapet, Hyderabad, TS, 500075, India

[2]Department of Mathematics, JNTUH College of Engineering Kukatpally, Hyderabad- 500085, TS, India

[3]Department of Mathematics, JNTUH College of Engineering Jagtial, Nachupally, Jagtial-505501,TS,India

Abstract:

The present paper, we presented the solution of the steady free convection boundary layer flow past an inclined plat plate embedded in a porous medium filled with nanofluids. In this model the non-dimensional equations are reduced into set of non-linear ordinary differential equations by using the similarity transformations which are solved by MATLAB in built numerical solver bvp5c. The effects of the velocity, temperature and nanoparticle fraction profiles are studied for different physical parameters like Brownian motion parameter Nb, the buoyancy-ratio parameter Nr, thermophoresis parameter Nt, Lewis number Le and the angle of inclination α through graphs. Also observed that variations of local Nusselt number with the Brownian motion parameter Nb, Sherwood number with the Brownian motion parameter Nb discussed graphically.

Keywords: Inclined flat plate, nanofluid, porous medium, thermophoresis, free convection Darcy-Boussinesq model, Brownian motion, Angle of inclination.

Introduction:

Convective flow in permeable media has been intensively considered in the on-going years because of its wide applications in designing as post accidental heat evacuation in atomic reactors, heat exchangers, drying forms, geothermal and oil recovery, building development, etc. Further, it is outstanding that conventional, including oil, water, and ethylene glycol blend are poor heat transfer fluid flows, since the thermal conductivity of these fluids plays an important role between the heat transfer medium and surface. Partha et.al [1] studied the viscous dissipation effects on the exponentially stretching surface of the mixed convection heat transfer flow. Rudraswamy et.al [2] analysed the Heat Transfer of a Nanofluid over an Exponentially Stretching Sheet in the presence of the chemical reaction and thermal radiation effects on MHD boundary layer flow. An analytical study of the thermal radiation effects on MHD boundary layer flow and heat transfer towards a porous exponentially stretching sheet are presented by Kalpna Sharma et.al [3]. Bhattacharyya and Layek [7] have investigated on the permeable stretching sheet with suction or blowing of the MHD boundary layer flow in the presence of the chemical reaction. Radiation and chemical reaction effects showed by using the finite difference method technique on the MHD flow of a vertical oscillating plate was presented by Shankar Goud[8]. Wang et.al [9] discussed the comprehensive model for the enhanced thermal conductivity of Nano fluids. BalReddy et.al [10] analysed the some effects are presented the magnetohydrodynamic boundary layer flow of nanofluid over an exponentially stretching permeable sheet using the Keller box solution technique. Bhadauria and Agarwal [11] discussed on natural convection in a nanofluid saturated rotating porous layer. Heat transfer of a nanofluid past a convectively heated stretching/shrinking sheet and Buoyancy effects on MHD stagnation point flow results are presented by Makinde et.al [12].

Mahendar et.al [13] studied on unsteady MHD flow past an accelerated vertical Plate of the Thermo- diffusion and diffusion -thermo effects with viscous dissipation-finite element study MHD stagnation-point flow past a stretching/shrinking sheet in the presence of heat generation/absorption and chemical reaction effects was studied by Freidoonimehr et.al [14]. Dharmendar Reddy et.al [15] discussed on MHD boundary layer flow of nanofluid and heat transfer over a porous exponentially stretching sheet in presence of suction and chemical reaction with thermal radiation effects. Heat generation/absorption on MHD stagnation flow of nanofluid towards a porous stretching sheet with prescribed surface heat flux was analysed by Jalilpour et.al [16]. Thirupathi et.al [17] studied on MHD natural convective flow of nanofluids past stationary and moving inclined porous plate considering temperature and concentration gradients with suction. Sheikholeslami [18] studied a numerical investigation of magnetic nanofluid free convective heat transfer in presence of variable magnetic field using two phase model. Pal and Mandal [19] Influence of thermal radiation on mixed convection heat and mass transfer stagnation-point flow in nanofluids over stretching/shrinking sheet in a porous medium with chemical reaction. Khan and Pop I [20] studied on Boundary-layer flow of a nanofluid past a stretching sheet. Flow and heat transfer characteristics on a moving plate in a nanofluid was studied by Bachok et.al [21]. Rahman et.al [22] discussed the Hydromagnetic slip flow of water based nanofluids past a wedge with convective surface in the presence of heat generation (or) absorption. Flow and heat transfer of a nanofluid over a nonlinearly stretching sheet: a numerical study was analysed by Rana and Bhargava [23]. Heat and mass transfer analysis for boundary layer stagnation-point flow towards a heated porous stretching sheet with heat absorption/generation and suction/blowing was reported by Layek et al [24]. Aly and Vajravelu [25] presented an Exact and numerical solutions of MHD nano boundary-layer flows over stretching surfaces in a porous medium. Convective heat transfer in the flow of viscous Ag-water and Cu-water nanofluids over a stretching surface studied by Vajravelu et.al [26].

The this paper, we study the solution free convection boundary layer flow past an inclined plat plate embedded in a porous medium filled with nanofluids. Set of coupled non-linear ordinary differential equations are solved by MATLAB in buit-numreical solver bvp5c and results of the velocity, temperature and nanoparticle fraction profiles are presented graphically.

Basic Equations:

Consider the steady free convection boundary past an inclined flat plate embedded in a porous medium filled with a nanofluid of ambient temperature. We presently make the standard boundary layer conditions, based on the scale analysis, and compose the Darcy-Boussinesq governing equations are follow

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \qquad \ldots (1)$$

$$\frac{\mu}{K}\frac{\partial u}{\partial y}=\mp\left[(1-C_{\infty})g\rho_{f\infty}\frac{\partial T}{\partial x}-g\left(\rho_p-\rho_{f_\infty}\right)\frac{\partial C}{\partial x}\right]\cos\alpha \qquad \ldots (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_m\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2\right] \quad \ldots (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\left(\frac{D_T}{T_\infty}\right)\frac{\partial^2 T}{\partial y^2} \quad \ldots (4)$$

Where x and y are the Cartesian coordinates measured along the inclined plate and normal to it, respectively to it, u and v are the velocity components along with x and y axes. Temperature and nanoparticle volume fraction of the fluid is denotes T & C, $\tau=\varepsilon(\rho C_p)_p/(\rho C_p)_f$ is a parameter. Here the $-$ & $+$ signs denote to the case of heated and cooled plate facing upward and downward.

The boundary conditions of equations. (1) – (4) are

$$v=0,\ T=T_w,\ C=C_w \quad at\ y=0$$
$$u=0,\ T=T_\infty,\ C=C_\infty \quad at\ y\to\infty \quad \ldots (5)$$

By using the stream function the velocity components are defined as in the usual manner

$u=\frac{\partial\psi}{\partial x}, v=-\frac{\partial\psi}{\partial y}$. Which can be extracted as $u=\alpha_m\frac{(Ra_x)^{\frac{2}{3}}}{x}f',\ v=-\alpha_m\frac{(Ra_x)^{\frac{1}{3}}}{3x}(f-2\eta f')$

using the following form of the similarities $\psi=\alpha_m(Ra_x)^{\frac{1}{3}}f(\eta),\eta=(Ra_x)^{\frac{1}{3}}\frac{y}{x}$,

$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}$, $\phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}$,where $Ra_x=\frac{(1-C_\infty)gK\beta(T_w-T_\infty)x}{\alpha_m v}$ is the local Rayleigh number and ψ is the stream function. On substituting the above similarities in eqns(2)-(4), we get the following ordinary differential equations for the boundary layer above a heated (or below a cooled) plate, here prime denotes the differentiation with respect to η.

$$f''-\frac{2}{3}\eta(\theta'-Nr\phi')\cos\alpha=0 \quad \ldots (6)$$

$$\theta''+\frac{1}{3}f\theta'+Nb\theta'\phi'+Nt\theta'^2=0 \quad \ldots (7)$$

$$\phi''+\frac{Le}{3}f\theta'+\frac{Nt}{Nb}\theta''=0 \quad \ldots (8)$$

the boundary conditions are

$$f(0)=0,\quad \theta(0)=1,\quad \phi(0)=1 \qquad \eta\to 0$$
$$f(\infty)=0,\quad \theta(\infty)=1,\quad \phi(\infty)=1 \quad \eta\to\infty \quad \ldots (9)$$

The non-dimensional parameters of the above equation (6) – (9) are defined as

$Nr=\frac{(\rho_\infty-\rho_{f_\infty})(C_w-C_\infty)}{(1-C_\infty)\rho_{f_\infty}\beta(T_w-T_\infty)}$ (buoyancy-ratio parameter),

$Le=\frac{v}{D_B}$ (Lewis number)

$Nt=\frac{(\rho C)_p D_T(T_w-T_\infty)}{(\rho c)_f vT_\infty}$ (Thermophoresis parameter)

$Nb=\frac{(\rho C)_p D_B(C_w-C_\infty)}{(\rho c)_f v}$ (Brownian motion parameter), When Nb and Nt are zero in equations (7)-(8) this boundary value issue diminishes to that of Cheng and Chang [3] for the instance of free convection boundary layer flow past a flat surface embedded in a Darcian liquid. It facts citing that Merkin and Zhang [4] have demonstrated that for the opposite case of the flow below heated (or over a cooled) impermeable plate, Equation (6) is replaced by

$$f'' + \frac{2}{3}\eta(\theta' - Nr\phi')\cos\alpha = 0 \qquad \ldots (10)$$

subject to the boundary conditions (9). Therefore, Merkin and Zhang [4] have demonstrated that there is no physically acceptable solution for the flat plate ($(\cos\alpha = 1)$ of equation (10) with the boundary conditions (9) so we won't manage this case. For the steady free convection of a Newtonian liquid (a nonporous medium), a boundary layer solution does not exist for the flow below a heated (or over a cooled) isothermal horizontal plate was established by Stewartson [5]. In this investigation, are the nearby Nusselt number Nu and the neighborhood Sherwood number Sh, which can be shown to be given by

$$(Ra_x)^{\frac{-1}{3}} Nu = -\theta'(0) \quad (Ra_x)^{\frac{-1}{3}} Sh = -\phi'(0) \qquad \ldots (11)$$

Results and Discussion

In order to study the natural convection boundary layer flow past a inclined vertical flat plate is studied in a porous medium filled with a nanofluid. The transformed set of coupled non-linear ordinary differential equations is solved by using the MATLAB in-built numerical solver bvp5c. In this situation when the heated plate is opposing upward (or below a cooled plate) is studied in this investigation. Figure 1 shows that the results of the buoyancy ratio and Brownian motion parameters Nr and Nb on the dimensionless velocity profiles, where the thermophoresis parameter, the Lewis number and angle of inclination $\mathrm{Nt} = 0.1, \mathrm{Le} = 0.5$ and $\alpha = \pi/6$ are kept constant. It shows that the velocity profiles decrease with decrease the Brownian motion parameter Nb with an increase in the buoyancy-ratio parameter Nr. The impacts of Nr and Nb on the dimensionless temperature profiles are shown in Figure 2 in this case it shows that the temperature profiles meet rapidly for smaller estimations of Nr. The heated boundary layer increments with increase in Nr. This is appeared in Fig. 2 when the heated plate is heat upward (or cooled below the plate). For this situation, the temperature profiles increment with an increase in Nr, while it diminishes with an increase in Nb.

When the heated plate is heated upward (or cooled downward), figure 3 shows that the dimensionless nanoparticle volume fraction profiles for different values of the buoyancy-ratio Nr and Brownian motion parametersNb. It shows that the nanoparticle fraction decrease with an increase in Nb but increase with an increase in Nr.

Figure 4 and 5 shows that the effects of the Lewis number Le and the Brownian motion parameter Nb on the dimensionless velocity and temperature profiles. It can be seen that at the velocity and dimensionless temperature profiles decrease with an increase in Le but increase with an increase in Nr and the remaining parameters kept at constant. When the heated plate is heated upward (or cooled downward),

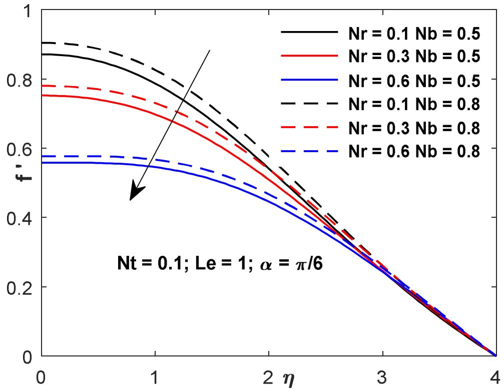

Fig. 1: Effect of Nr and Nb on dimensionless velocity.

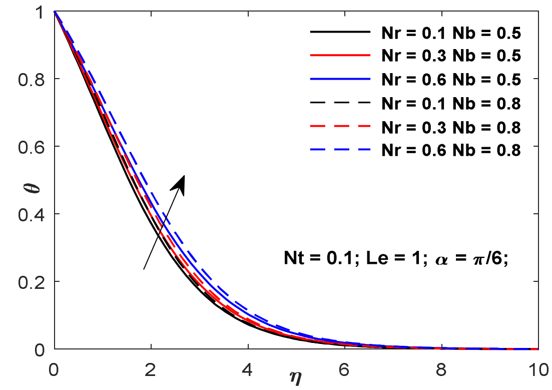

Fig. 2 Effect of Nr and Nb on dimensionless temperature profiles

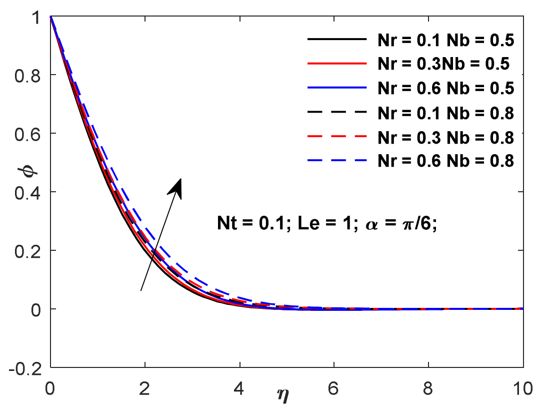

Fig. 3: Effect of Nr and Nb on dimensionless nanoparticle volume fraction profiles.

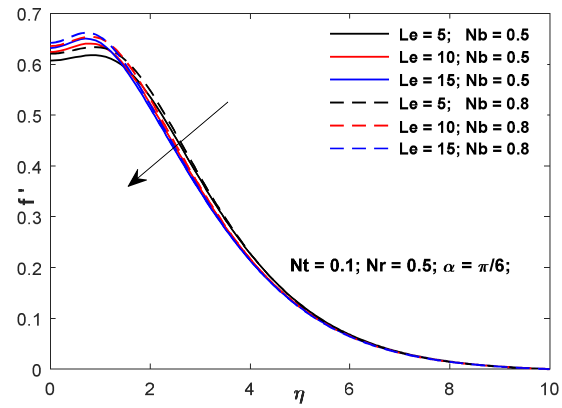

Fig.4: Effects of Le and Nb dimensionless velocity profiles

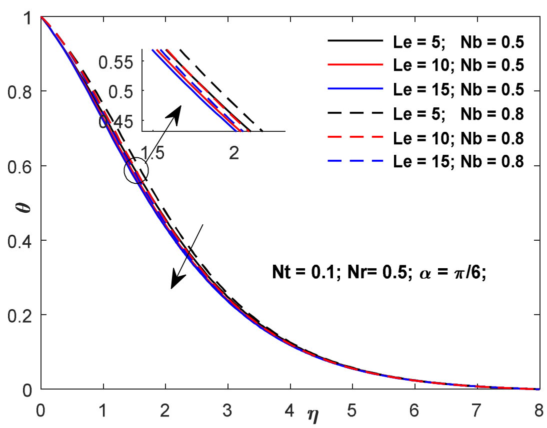

Fig.5:Effects of Le and Nb on the dimensionless temperature profiles

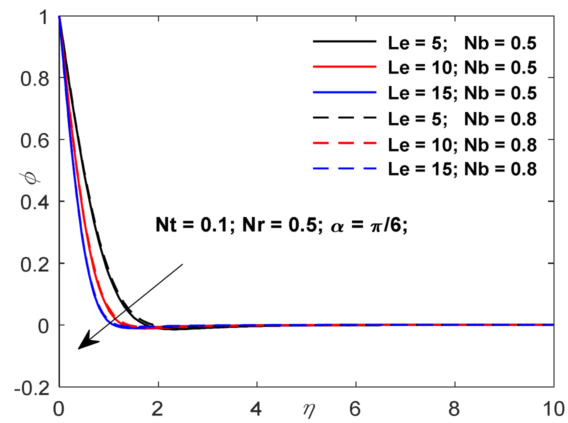

Fig.6: Effects of Le and Nb dimensionless nanoparticle volume fraction profiles

The effects of Le and Nb on the dimensionless nanoparticle volume fraction profiles are shown in figure 6. It is observed that the nanoparticle volume fraction profiles decrease with an increase in Le but increase with an increase in Nb . Fig. 7 shows that the dimensionless velocity depends on the Nb and the angle of inclination. It is found that the minimum value of α for which the dimensionless velocity decreases with an increase in the angle of inclination but increase with an increase of Nb.

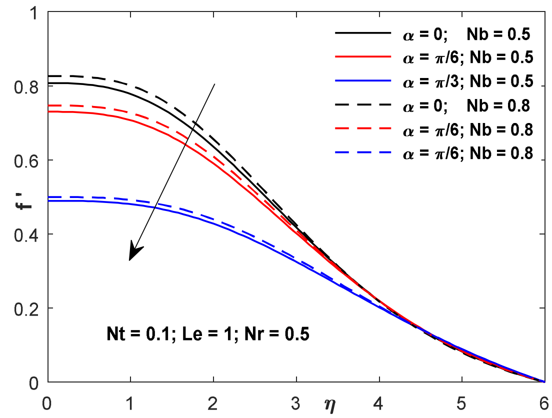

Fig.7: Effects of α and Nb on the dimensionless velocity profile

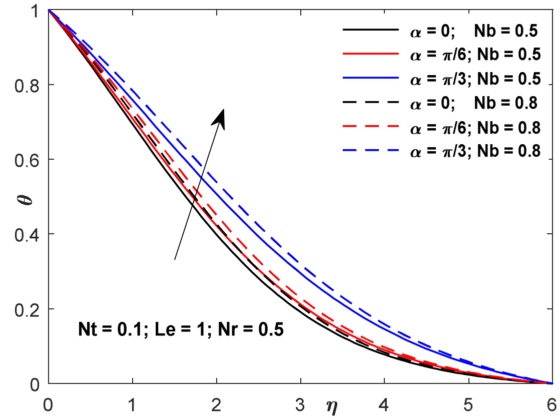

Fig.8: Effects of α and Nb temperature profile

The effects of α and Nb on the dimensionless temperature and Nanoparticle volume fraction profiles are shown in Figure 8 an 9. In this case, It is found that that the temperature and Nanoparticle volume fraction profiles are increases with an increase in α and Nb. heated plate is facing upward (or cooled below the plate).

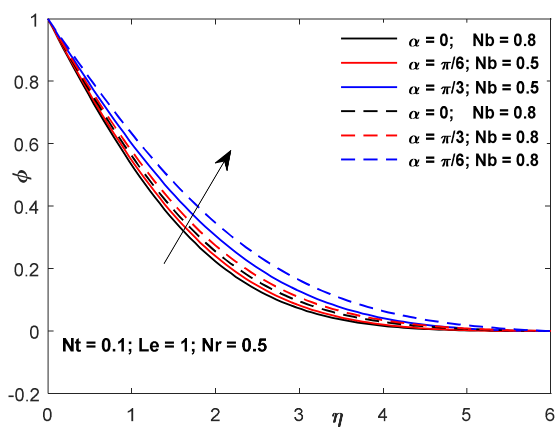

Fig.9: Effects α and Nb on the dimensionless Nanoparticle volume fraction profiles

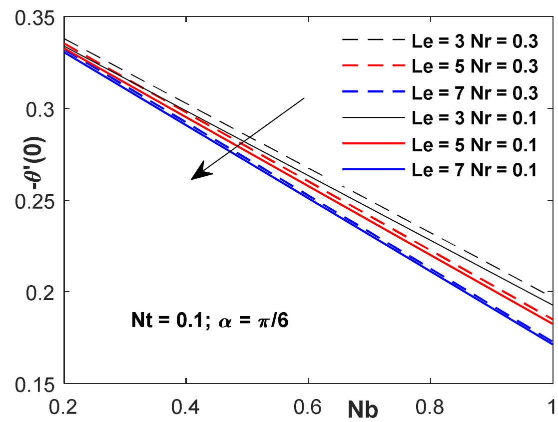

Fig10: Variation $-\theta'(0)$ and Nb when the plate is heated upward or cooled downward

Figure 10 describes the variations of the local Nusselt number with Nb. When the heat plate is facing upward (or cooled descending). It is seen that the local Nusselt number decreases with an expansion in the parameters Nr and Nb for different values of Lewis number Le. The impacts of Nr on the local Nusselt number are also sen in figure10. It is observed that the local Nusselt number decreases with an expansion in Nr. The variations of the local Sherwood number with Nb is appeared in Figure11 for various estimations of the Lewis number Le. The heatd plate is confronting upward. It is obvious from assume that the local Sherwood numbers increment with an expansion in Nt and decrease with an expansion in Nb. The local Sherwood numbers diminish with an expansion in Lewis numbers, as appeared in Figure.11.

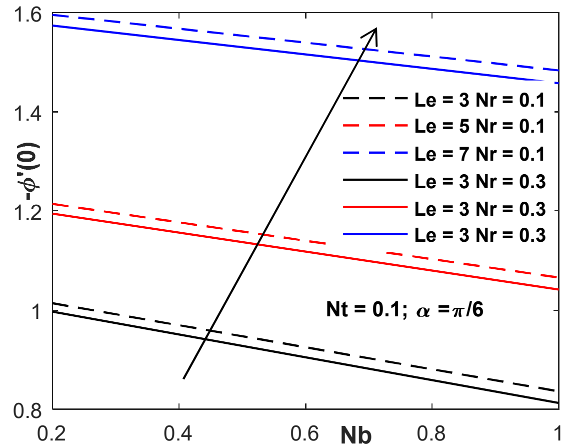

Fig.11: Variation of the local Nusselt number with the Brownian motion parameter Nb

Conclusions

In this paper we investigate the governing equations are solved by employing MATLAB in built numerical solver bvp5c. The results of the flow parameters on the velocity, temperature, nanoparticle volume fraction profiles, local Nusselt and Sherwood numbers are investigated. Some of the important conclusions of this study are as follows:

- Velocity decreases with decrease Nb with an increase in Nr.
- Temperature and nanoparticle fraction increases with an increase in Nr,while it diminishes with an increase in Nb.
- Velocity and temperature decrease with an increase in Le but increase with an increase in Nr
- Nanoparticle volume fraction profiles decrease with an increase in Le but increase with an increase in Nb .
- Velocity decreases with an increase in the angle of inclination but increase with an increase of Nb.
- Temperature and Nanoparticle volume fraction increases with an increase α and Nb.
- Local Nusselt number decreases with an increase of Nr and Nb for different values of Le.
- Local Sherwood numbers increment with increase in Nt and decreases Nb.

REFERENCES:

1. M. K. Partha, P. V. S. N. Murthy, and G. P. Rajasekhar, *Int*.J *Heat and Mass Transfer,* 41(4), pp. 360-366, 2005.
2. N. G. Rudraswamy, B. J. Gireesha., *J. of App. Math and Physics*, 2, pp.24-32, 2014.
3. Chang, I.-D., and Cheng, P., *Int. J. Heat Mass Transfer*, 26, pp. 163–173, 1983.
4. Merkin, J. H., and Zhang, G., *Wärme - und Stoffübertragung*. 25, pp. 179–184, 1990.
5. Stewartson, K. *J. Appl. Math. Phys*.,9, pp. 276-28, 19581.
6. Kalpna Sharma., Sumit Gupta, *Int. J. Adv.Appl.Math.and Mech*.4(1) ,pp.1- 10 , 2016
7. K. Bhattacharyya and G. C. Layek, *Chemical Engineering Communications*, 197(12), pp. 1527-1540,2010.

8. B. Shankar Goud., *Int.J. of Emerging Tech. in Engg Research (IJETER),* 5(11), pp.32-35, 2017.
9. W. Wang, L. Lin, Z. X. Feng, S. Y.Wang., *J.of Adv. Research in Physics*, 3(2), Article ID 021209, 2012
10. G. Bal Reddy., B. Shankar Goud.,MN. Raja Shekar., *Int.J.of Mech. Engineering and Technology,* 9(10), pp. 1646–1656, 2018.
11. B. S. Bhadauria and S. Agarwal,*Transport in Porous Media*, 87(2), pp.585-602, 2011.
12. O.D. Makinde, W.A. Khan, Z.H. ., *Int. J. Heat Mass Transf.*, 62, pp. 526-533, 2013.
13. D. Mahendar., B. Shankar Goud and P.SrikanthRao., *Int. Journal of Pure and applied mathematics*, 120 (6), pp.8165-8185, 2018.
14. N Freidoonimehr, MM Rashidi, B., *J Brazil Soc Mech Sci Eng* pp. 1-10, 2015, DOI: 10.1007/s40430-015-0456-8.
15. Y.Dharmendar Reddy., V.Srinivasa Rao, .Anand Babu., *Int J. of Maths Trends and Technology (IJMTT),* 47 (2) , pp.87-100, 2017
16. B. Jalilpour, S. Jafarmadar, D.D. Ganji, A.B. Shotorban, H. Taghavifar., *J Mol Liq,* 195 , pp. 194-204, 2014.
17. Thirupathi Thumma, A Chamkha, Siva Reddy Sheri., *Int.J of Numerical Methods for Heat & Fluid Flow* , 27(3), Pages 1765-1794,2017.
18. M. Sheikholeslami, M.M. Rashidi, D.D. Ganji., J Mol Liq, 212.,pp. 117-126, 2015.
19. D. Pal, and G. Mandal., *Nuclear Eng. Design*, 273, pp. 644-652, 2014.
20. Khan WA, Pop I., *Int J Heat Mass Transf,* 53, pp.2477-2483, 2010.
21. Bachok, N., Ishak, A., & Pop I., *Int. Journal of Heat and Mass Transfer,* 55, pp. 642-648, 2012.
22. Rahman, M. M., Al-Lawatia, M. A., Eltayeb, I. A., & Al-Salti, N., *Int.J. of Thermal Sciences,* 57,pp 172–182, 2012.
23. P. Rana and R. Bhargava, *Comm. in Nonlinear Science and Numerical Simulation*, 17(1), pp. 212-226, 2012.
24. G. C. Layek, S. Mukhopadhyay, and S. K. A. Samad, *Int. Comm. in Heat and Mass Transfe*r, 34(3), pp. 347-356, 2007.
25. E. H.Aly.,K.Vajravelu,*Applied Mathematics and Computation*,232,pp.191-204, 2014.
26. K. Vajravelu, K. V. Prasad, J. Lee, C. Lee, I. Pop, and R. A. van Gorder, *Int.Journal of Thermal Sciences*, 50(5), pp. 843–851, 2011.

Nomenclature			
ε	porosity	g	Gravitational acceleration
u, v	Velocity components along x, y directions	μ	Dynamic viscosity
η	Similarity variable	ν	Kinematic viscosity
T	Temperature	β	Volume thermal diffusivity coefficient
C	Nonoparticle volume fraction	α_m	Effective thermal diffusivity
K	Permeability of the porous medium	D_B	Brownian diffusion coefficient Thermophoretic
D_T	Thermophoretic diffusion coefficient	T_w & C_w	Temperature & Nanoparticle volume fraction at the plate
T_∞ & C_∞	Ambient Temperature & Nanoparticle volume fraction		