Performance of PAPR, BER, SER, PER for Multi-User MIMO-OFDM Downlink Scheme and MU Precoding Schemes

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Abstract

The demand for high-quality communication services, anywhere, anytime, is growing rapidly. It is a great challenge for system designers to deal with techniques that improve the provided Quality of Service and reduces the effect of delay spread, Doppler shift. To fulfill this requirement, Orthogonal Frequency Division Multiplexing (OFDM) is an attractive technology, as it deals with the multicarrier modulation. It has several advantages like high data rate transmission with high bandwidth efficiency, exceptional robustness to multi-path fading and low implementation complexity. However, the OFDM system has limitations. The OFDM signal is formed from the linear addition of independently modulated subcarriers. As a result, when the number of subcarriers is large, high amplitude peaks occur which results in high Peak-to-Average Power Ratio (PAPR). Usually, the OFDM signal is passed through a non-linear power amplifier before its transmission over the channel. High PAPR value requires complex high power amplifier (HPA) operating on a very large linear range at the transmitter. Otherwise, the nonlinearity of the HPA leads to in-band distortion, which increases the bit error rate (BER) of the system, and the out-of-band distortion, which introduces high adjacent channel interference. different methods are compared reducing the PAPR, Bit Error, Symbol Error Rate and Packet Error Rate in MIMO-OFDM systems.

Keywords: OFDM, MIMO, PAPR, BER, SER, PER.

1. Introduction

Multiple-input multiple-output (MIMO) wireless communication is useful to the next generation for increasing high data rates and quality-of-services and to give higher throughput[2]. A large number of antennas placed at ground-station (GS) would provide a large number of users and in the same frequency band, but with the number of GS antennas being much higher than the number of users[3], say ten users used a hundred antennas. MIMO systems also have the possible to decrease operational power using at the transmitter and enable the use of low-complexity methods for suppressing MU interference (MUI). All these properties render MIMO a promising technology for next-generation wireless communication systems.
While the theoretical aspects of MU-MIMO systems have gained significant attention in the research Manuscript community e.g.,[2]-[6], much less is known about practical transmission schemes. As pointed out in [7], Low-cost and low-power radio-frequency (RF) components will be required for the MIMO systems. For the frequency-flat channels, multi-user pre-coding method is proposed, which depends on per-antenna constant envelope (CE) transmission to enable suitable implementation using non-linear RF components. Moreover, which is not necessarily optimal as in practice there is always a trade-off between PAR, error-rate performance, and power amplifier efficiency. Practical wireless channels typically show the frequency selective fading and a low-PAR precoding solution right for such channels would be desirable[7]. Preferably, the solution should be such that the complexity needed in each (mobile) terminal is small (due to the difficult area and power constraints), although, heavier processing could be provided at the GS.

Orthogonal frequency-division multiplexing (OFDM) [8] is a well-established and efficient way of dealing with the frequency selective channels. In addition to simplifying the equalization at the receiver, OFDM also facilitates per-tone power and bit allocation, scheduling in the frequency domain, and spectrum shaping. However, OFDM is suffering from a high PAPR which required the use of linear RF components (e.g., power amplifiers) to reduce out-of-band radiation (OBR) and signal distortions. Unfortunately, linear RF components are, in general, more costly and less power efficient than their non-linear counterparts, which would eventually result in exorbitant costs for large-scale BS implementations having hundreds of antennas. Therefore, it is of paramount importance to reduce the PAR of OFDM-based-MIMO systems to facilitate corresponding low-cost and low-power BS implementations. To overcome challenging requirements and problems of OFDM has been different types of PAR-reduction schemes for point-to-point for single-antenna and MIMO wireless systems For MU-MIMO systems[9].

A) Notation

uppercase boldface letters designate matrices and Lowercase boldface letters stand for column vectors and. For a matrix X, we denote its transpose, conjugate transpose, and largest singular value by $X^T$, $X^H$ and $\mu_{\text{max}}(X)$, respectively; $X^\dagger = X^H(XX^T)^{-1}$ stands for the pseudo-inverse of X and the entry in the $i$th row and $j$th column is $[X]_{i,j}$. The Q×Q identity matrix is denoted by IQ, the Q×N all-zeros matrix by $0_{Q \times P}$, and FM refers to the Q × Q discrete Fourier transform (DFT) matrix. The $j$th entry of b vector a is designated by $b_k$; the Euclidean (or $j_2$) norm is denoted by $||b||_2 = \max_j |b_j|$ stands for the $j_\infty$ -norm and the $j_\infty$ -norm is defined as $||b||_\infty = \max_j (|u_b||_\infty , |v_b||_\infty \} with u_b, v_b$ representing the real and imaginary part of b, respectively. Sets are designated by upper-case calligraphic letters; the cardinality and complement of the set is $|T|$ and $T^c$, respectively. For $x \in V$ we define $[x]^+ = \max\{x, 0\}.$
2. System model

Let’s consider an OFDM-based MU-MIMO downlink scenario as depicted in Fig. 1. The BS is assumed to have a significantly larger number of transmit antennas P than the number Q<<N of independent terminals (users); each terminal is equipped with a single antenna only. The signal vector \( c_w \in \mathbb{C}^Q \) contains information for each of the Q users, where \( w = 1, \ldots, W \) indexes the OFDM tones, \( W \) corresponds to the total number of OFDM tones, ‘O’ represents the set of scalar complex-valued constellations, and \( [[c_w]]_q \in \mathbb{C}^Q \) corresponds to the symbol at tone \( w \) to be transmitted to user \( q \). To normalize the symbols to satisfy \( \mathbb{E}\{|[c_w]]_q|^2\} = 1/Q \) to shape the spectrum of the transmitted signals, specify certain unused tones specify by the OFDM systems (spectrum at both ends). Hence, we set \( c_w = 0_{Q \times 1} \) for \( w \in T^C \) where \( T \) designates the set of tones used for data transmission. In order to remove MUI, the signal vectors \( c_w, \forall w \) are passed through a precoder, which generates \( W \) vectors \( x_w \in \mathbb{C}^N \) according to a given precoding scheme (see Section II-B). Since precoding causes the transmit power \( P = \sum_{w=1}^{W} |A_w|^2 \) to depend on the signals \( c_w, \forall w \) and the channel state, we normalize the precoded vectors \( A_w, \forall w \) prior to transmission as

\[
A_w = A_w/\sqrt{\sum_{w=1}^{W} |A_w|^2} \quad \text{w} = 1, \ldots, W,
\]

Which confirms unit transmit power. To emphasize that this normalization is an essential step in practice (i.e., to meet regulatory power constraints). To simplify the presentation, however, the normalization is omitted in the description of the precoder (but normalization is employed in all simulation results shown in Section V). Hence, in what follows \( A_w \) and \( \hat{A}_w \) are treated interchangeably. The (normalized) vectors, \( A_w \) are then reordered (from user orientation to transmit-antenna orientation) according to the following one-to-one mapping:

![Figure 1. MIMO-OFDM downlink](image_url)
\[ [A_1 \ldots A_W] = [x_1 \ldots x_N]^T \]  

Here, the W-dimensional vector corresponds to the (frequency-domain) signal to be transmitted from the nth antenna. The time-domain samples are obtained by applying the inverse DFT (IDFT) according to \( \tilde{x}_n = F_W^H a_n \) followed by parallel-to-serial (P/S) conversion. Prior to modulation and transmission over the wireless channel, a cyclic prefix (CP) is added to the (time-domain) samples \( \tilde{x}_n \) for \( n \) to avoid ISI. To simplify the explanation, to identify the input-output relation of the wireless channel in the frequency domain. Concretely, we consider

\[ y_w = H_w a_w + n_w \quad w=1,2\ldots W \]  

where \( y_w \) denotes the wth receive vector, \( H_w \in \mathbb{C}^{Q \times N} \) represents the MIMO channel matrix associated with the wth OFDM tone, and \( N_0 \) is an M-vector of i.i.d. complex Gaussian noise with zero-mean and variance \( N_0 \) per entry. The average receive signal-to-noise ratio (SNR) is defined by \( \text{SNR} = 1/N_0 \). Finally, each of the M user terminals performs OFDM demodulation to obtain the received (frequency domain) signals \( [y_w]_m \) for\( w = 1, \ldots , W \) (from Figure. 1).

A. MU Precoding Schemes

In order to avoid MUI, precoding must be employed at the GS. To this end, we assume the channel matrices \( H_w, \forall w \) to be known perfectly at the transmit-side. Linear precoding now amounts to transmitting \( a_w = G_w c_w \), where \( G_w \in \mathbb{C}^{N \times Q} \) is a suitable precoding matrix. One of the most prominent precoding schemes is least-squares (LS) precoding (or linear zero-forcing precoding), which corresponds to \( G_w = H_w^\dagger \). Since \( H_w H_w^\dagger = I_M \) transmitting \( a_w = H_w^\dagger c_w \) perfectly removes all MUI, i.e., it transforms (3) into M independent single-stream systems \( y_w = c_w + n_w \). Note that LS precoding is equivalent to transmitting the solution \( \cdot a_w \) to the following convex optimization problem:

\[ \text{(LS) minimize} \| \hat{x} \|_2 \text{ subjected to } c_w = H_w \hat{x} \]

This formulation inspired us to state the MU-MIMO-OFDM downlink transmission scheme proposed in Section III as a convex optimization problem. Several other linear precoding schemes have been proposed in the literature, such as matched-filter (MF) precoding, minimum-mean-square-error (MMSE) precoding [17], or more sophisticated non-linear schemes, such as dirty-paper coding. In the remainder of the paper, we will occasionally consider MF precoding, which corresponds to \( G_w = H_w^H \). Since \( H_w H_w^H \) is in general, not a diagonal matrix, MF is normally unable to remove the MUI. Nevertheless, MF precoding was shown in [6] to be competitive for large-scale MIMO in some operating regimes and in [3] to perfectly remove MUI in the large-antenna limit, i.e., when \( N \to \infty \).

B. peak to average power ratio

The IDFT required at the transmitter causes the OFDM signals \( \tilde{x}_n \), \( \forall n \) to exhibit a large dynamic range [8]. Such signals are susceptible to non-linear distortions (e.g. saturation or clipping) typically induced by real-world RF components. To avoid undesirable out-of-band radiation and signal distortions totally, linear RF components and PAPR-reduction methods are key to successfully install OFDM in practical systems.
1) PAR Definition: The dynamic range of the transmitted OFDM signals is typically characterized through the peak-to-average (power) ratio (PAR). Since many real-world RF-chain implementations process and modulate the real and imaginary part independently, we define the PAR at the nth transmit antenna as

$$\text{PAPR} = \frac{2W||x_n||_\infty^2}{||x||_\infty^2} \quad (4)$$

As a consequence of standard vector-norm relations, (4) satisfies $1 \leq \text{PAPR} \leq 2W$. Here, the upper bound corresponds to the worst-case PAR and is achieved for signals having only a single (real or imaginary) non-zero entry. The lower bound corresponds to the best case and is realized by transmitting vectors whose (real and imaginary) entries have constant modulus. To minimize distortion due to hardware non-linearity, the transmit signals should have a PAR that is close to one; this can either be achieved by CE transmission [7] or by using sophisticated PAR-reduction schemes.

3. Downlink transmission scheme

The main idea of the downlink transmission scheme developed next is to jointly perform MU pre-coding, OFDM modulation, and PAR reduction, by exploiting the DoF available in large-scale MU-MIMO systems. To convey the basic idea and to characterize its fundamental properties, we start by considering a simplified MIMO system. We then present the MU-MIMO-OFDM downlink transmission scheme in full detail and conclude by discussing possible extensions

Joint Pre-coding, Modulation, and PAR Reduction (PMP):

The application of (P-INF) to each time-domain sample after OFDM modulation would reduce the PAR but, unfortunately, would no longer allow the equalization of ISI using conventional OFDM demodulation. In fact, such a straightforward PAR-reduction approach would necessitate the deployment of sophisticated equalization schemes in each terminal. To enable the use of conventional OFDM demodulation in the receiver, we next formulate the convex optimization problem, which jointly performs MU pre-coding, OFDM modulation, and PAR reduction.

In order to remove MUI, the following pre-coding constraints must hold:

$$c_w = H_w a_w, \ w \in T \quad (5)$$

To ensure certain desirable spectral properties of the transmitted OFDM signals, the inactive OFDM tones (indexed by $T^c$) must satisfy the following shaping constraints:

$$0_{N \times 1} = a_w, \ w \in T^c \quad (6)$$

PAR reduction is achieved similarly to (P-INF), with the main difference that we want to minimize the $f_{\infty}$-norm of the time-domain samples, $\tilde{x}_n$, $\forall n$. In order to simplify notation, we define the (linear) mapping between the time-domain samples $\tilde{x}_n$, $\forall n$, and the wth (frequency-domain) transmit vector $a_w$ as $a_w = f_w (\tilde{x}_1, \ldots, \tilde{x}_N)$, where the linear function $f_w(\cdot)$ applies the DFT according to $x_n = F_w \tilde{x}_n$, $\forall n$ and performs the re-ordering defined in (2).

With (5) and (6), we are able to formulate the downlink transmission scheme as a convex optimization problem:

$$\begin{align*}
\text{minimize} & \quad \tilde{x}_1, \ldots, \tilde{x}_N \ 	ext{max}\{|\tilde{x}_1|, |\tilde{x}_2|, \ldots, |\tilde{x}_N|\} \\
\text{subjected to} & \quad c_w = H_w f_w (\tilde{x}_1, \ldots, \tilde{x}_N), \ w \in T
\end{align*}$$

(PMP)
\[ 0_{N \times 1} = f_w(\bar{x}_1, \ldots, \bar{x}_N), \quad w \in T^c \quad (7) \]

The vectors \( \bar{x}_n, \forall n \) which minimize (PMP) correspond to the time-
domain OFDM samples to be transmitted from each antenna. Following the reasoning of
Section III-A, we expect these vectors to have low PAR (see Section V for corresponding
simulation results). In what follows, “PMP” refers to the general method of jointly
performing pre-coding, modulation, and PAR reduction, whereas “(PMP)” refers to the
actual optimization problem stated above.

4. Simulations results

In this section, the efficacy of the proposed joint precoding, modulation, and PAR
reduction approach is demonstrated and provide a comparison to conventional MU
precoding schemes.

A. Simulation Parameters

Unless explicitly stated otherwise, all simulation results are for an MU-MIMO-
OFDM system having \( N = 100 \) antennas at the BS and serving \( M = 10 \) single-antenna
terminals. We employ OFDM with \( W = 128 \) tones and use a spectral map \( T \) as specified in
the 40MHz-mode of IEEE 802.11n [20]. We consider coded transmission, i.e., for each
user, we independently encode 216 information bits using a convolutional code (rate-1/2,
generator polynomials, and constraint length 7), apply random interleaving (across OFDM
tones), and map the coded bits to a 16-QAM constellation (using Gray labeling). To
implement (PMP-L), to use FITRA as detailed in Algorithm 1 with a maximum number of
\( K = 2000 \) iterations and a regularization parameter of \( \lambda = 0.25 \). In addition to LS and MF
precoding, also consider the performance of baseline precoding and PAR-reduction
method. To this end, we employ LS precoding followed by truncation (clipping) of the
entries of the time-domain samples \( a_n, \forall n \), use a clipping strategy where one can specify
a target PAR, which is then used to compute a clipping level for which the PAR in (4) of
the resulting time-domain samples is no more than the chosen target PAR. The precoded
and normalized vectors are then transmitted over a frequency-selective channel modelled
as a tap-delay line with \( T = 4 \) taps. The time-domain channel matrices \( b H_t, t = 1, \ldots, T \),
that constitute the impulse response of the channel, have i.i.d. circularly symmetric
Gaussian distributed entries with zero mean and unit variance. To detect the transmitted
information bits, each user \( m \) performs soft output demodulation of the received symbols
\([y]_m, w = 1, \ldots, W \) and applies a soft-input Viterbi decoder.
Figure 2. Time domain representation of different pre-coding schemes

Figure 3. Frequency domain representation of different pre-coding schemes
Figure 4. PAPR performance of various pre-coding schemes

Figure 5. SER performance of various pre-coding schemes
B. Performance Measures

To compare the PAPR characteristics of different precoding methods, we use the complementary cumulative distribution function (CCDF) defined as $\text{CCDF (PAPR)} = P\{\text{PAPR}_n > \text{PAPR}\}$. To furthermore define the “PAPR performance” as the maximum PAPR level $\text{PAPR}^*$ that is met for 99% of all transmitted OFDM symbols, i.e., given by $\text{CCDF (PAR}^*) = 1\%$. The error-rate performance is measured by the average (across users) symbol-error rate (SER); a symbol is said to be in error if at least one of the information bits per received OFDM symbol is decoded in error. The “SNR operating point” corresponds to the minimum SNR required to achieve 1% SER.
4. Conclusions
Figures 2 and 3 summarize the key characteristics of PMP and its PAR-reduction capabilities and error-rate performance are compared with those of LS and MF precoding, as well as to LS precoding followed by clipping (denoted by “LS+clip” in the following). Fig. 2 shows the real part of a time domain signal for all precoding schemes (the imaginary part behaves similarly). Clearly, PMP results in time-domain signals having a significantly smaller PAR than that of LS and MF; The frequency-domain results shown in Fig.3 confirm that LS, MF, and PMP maintain the spectral constraints. Fig. 4 shows the PAR-performance characteristics for all considered precoding schemes. One can immediately see that PMP reduces the PAR by more than 11dB compared to LS and MF precoding (at CCDF PAR = 1%); as expected, LS+clip achieves 4dB PAR deterministically.

Figure 5 shows the symbol error rate is PMP is better compared with LS+clip and MF, the LS performs better SNR compared with PMP due to signal normalization. From figure 6 shows the performance of BER is better than LS+clip, and MF. Figure 7 shows the PMP is better compared to the MF for packet error rate.

6. References


