Response of FGM Simply Supported Square Plate under Mechanical Load Subject to Variable Thermal Environment

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Abstract

A typical FGM, with a high bending strength is a non-homogeneous composite made from different phases of material constituents (usually ceramic and metal). Functionally graded materials (FGM) have extensively been used because of their attractive properties, including a potential reduction of in-plane and transverse through-the-thickness stress, enhanced thermal properties, etc. An FGM can be defined by the variation in the volume fractions. The effective material properties like Young's modulus, Poisson's ratio, coefficient of thermal expansion, thermal conductivity etc. on the upper and lower surfaces are different but are pre-assigned. Volume fraction and effective material properties of FGMs may vary in the thickness direction or in the plane of a plate. P-FGM follow the Power-law function and S-FGM follow Sigmoid law function to have smoother variation in properties as compared to Power law. With the increasing applications of functionally graded materials it is important to understand the behaviour of thermo-mechanical deformation response of FG plates. It is also important to study the response of the FG materials following various material gradient laws for properties. Keeping this in consideration the objective of the present work is to examine the thermo-mechanical behaviour for variable thermal environment, various material gradient laws and various volume fraction exponents. The results are presented in the form of non-dimensional parameters.

Keywords: *Functioally, ceramic, nondimensional, thermomechanical.*

1. Introduction

Functionally graded materials (FGM) have extensively been used because of their attractive properties, including a potential reduction of in-plane and transverse through-the-thickness stress, an improved residual stress distribution, enhanced thermal properties, etc. The functionally graded material (FGM) can be produced by gradually and continuously varying the constituents of multi-phase materials in a pre-determined profile. An FGM can be defined by the variation in the volume fractions. Volume fraction and effective material properties of FGMs may vary in the thickness direction or in the plane of a plate. The effective material properties like Young's modulus, Poisson's ratio, coefficient of thermal expansion, thermal conductivity etc. on the upper and lower surfaces are different but are pre-assigned. Volume fraction and effective material properties of FGMs may vary in the thickness direction or in the plane of a plate. Power-law function (P-FGM) is used to define volume fraction in FGM [1,2]. Sigmoid law function (S-FGM) is also used to define volume fraction in FGM so that have smoother variation in properties as compared to Power law and thereby the effective variation in material properties through the thickness.[3,4,5]. A typical

FGM, with a high bending strength is a non-homogeneous composite made from different phases of material constituents (usually ceramic and metal). Various combinations of ceramic and metals have been used and analyzed by the researchers e.g. Aluminum-Silicon Carbide [6], Aluminum-Alumina [7] and Aluminum-Zirconia [8,9]. To ascertain the distribution of stress and displacement for a plate subjected to a thermomechanical load requires a consideration of number of basic conditions. Neglecting the transverse shear deformations and using first order shear deformation theory is expected to give good results and the equations of motion based on the combination of the first order plate theory and the Von Karman strains has also been developed [10]. The through-thickness distributions of the in-plane displacements and transverse shear stresses in a functionally graded plate do not agree with those assumed in classical and shear deformation plate theories. It was reported that the assumption of constant deflection is not true for thermal load but it was found to be true for mechanical load [11]. For both static and dynamic loads, the centroidal deflection of a FGM plates lies between those for a pure ceramic and a pure metallic plate. It was concluded that the stresses in the FGM plate along the thickness direction were not linearly proportional to thickness [12]. Third-order theory and non-linear first-order theory has been used and at the same time Von-Karman type geometric non-linearity was taken into account [8]. For both static and dynamic loads, the centroidal deflection of a FGM plate was lying between those for a pure ceramic and a pure metal plate [1]. The assumption of constant deflection is not true for thermal load but it was found to be true for mechanical load [13]. The thermo-elasto-static and thermo-elasto-dynamic response of plates subjected to pressure loading and thickness varying temperature fields has been examined [14]. The stability and failure response of elastoplastic Ni/Al2O3 functionally graded plate under thermomechanical load using non-linear finite element formulation based on first-order shear deformation theory and von-Karman’s nonlinear kinematics has been explored [15]. A refined sinusoidal model considering transverse normal strain for thermoelastic analysis of functionally graded material plate has been developed [16].

With the increasing applications of functionally graded materials it is important to understand the behaviour of thermo-mechanical deformation response of FG plates. It is also important to study the response of the FG materials following various material gradient laws for properties. Keeping this in consideration the objective of the present work is to examine the thermo-mechanical behaviour for variable thermal environment, various material gradient laws and various volume fraction exponents. The results are presented in the form of non-dimensional parameters.

2. Thermal and thermomechanical analysis using finite element method

The thermo-mechanical analysis is conducted for FGM made of combination of metal and ceramic. The metal and ceramic chosen are Aluminum and Zirconia respectively. The Young’s modulus for Aluminium is 70 GPa and that for Zirconia is 151 GPa. The coefficient of thermal expansion for Aluminium is $23x10^{-6}$ /°C and that for Zirconia is $10x10^{-6}$ /°C. The Poisson's ratio for both the materials was chosen to be 0.3. The effect of Poisson’s ratio on the deformation is much less as compared to that of Young’s modulus. A square FGM plate simply supported at all of its edges (SSSS) is considered here. The thickness of the plate (h) is taken 0.02m and the aspect ratio is taken unity. The thermal analysis is performed by varying temperature. The temperature of ceramic top surface is varied from 50 C to 400 C. The lower metallic surface and all the edges are kept at a temperature of 0 C. The thermomechanical analysis is performed by applying uniformly distributed load (udl) with varying temperature. The value of the uniformly distributed load (udl) chosen is equal to $1X10^6$ N/m^2. The temperature of ceramic top surface is varied from 50 C to 400 C. The lower metallic surface and all the edges are kept at a temperature of 0 C. The analysis is performed for various values of the volume fraction exponent (n) in P-FGM and S-FGM. The results are presented in terms of non-dimensional parameters i.e. non-dimensional deflection ($\overline{u_z}=u_z/h$), non-dimensional tensile stress ($\overline{\sigma_x}=\sigma_x/p_o$) and non-dimensional shear stress ($\overline{\sigma_{xy}}=\sigma_{xy}/p_o$) where '$u_z$' is deflection, 'σ' is stress, 'a' and 'b' are side lengths of plate, and p_o is applied load (N/m^2). The material properties of the FGM vary throughout the thickness. The material properties are calculated using the Power law and Sigmoid law of volume fraction distribution. The thermal analysis of FGM plate is conducted using finite element model and the ANSYS software is being used for computing the thermal response.

3. Numerical results

The thermal response of FGM plate has been carried out for two conditions, namely: variable thermal environment and Variable thermal environment under constant mechanical load.

3.1 Variable thermal environment

In this section, the results of the analysis performed on a simply supported square (a/b=1) FGM plate subject to varying thermal environment are reported and discussed. The thermal analysis is performed by varying temperature of ceramic surface from 50°C to 400°C. The effect of various volume fractions and various laws i.e. P-FGM and S-FGM are studied. The results are presented in terms of non-dimensional parameters i.e. non-dimensional deflection ($\overline{u_z}$), non-dimensional tensile stress ($\overline{\sigma_x}$) and non-dimensional shear stress ($\overline{\sigma_{xy}}$).

a. Non-dimensional deflection ($\overline{u_z}$)

Figure 1 and Figure 2 show the effect of variation of temperature on non-dimensional deflection ($\overline{u_z}$) for simply supported plate in thermal environment for P-FGM and S-FGM respectively. The comparison of results for various values of volume fraction exponent 'n' for P-FGM and S-FGM has been presented.

The effect of variation of temperature of a simply supported square FGM plate reveals the following information:

(a) The non-dimensional deflection increases linearly with temperature. Because of the high temperature at the top surface, the plate is having upward deflection. The deflection of the metal plate is maximum because the metal has higher coefficient of thermal expansion.

(b) The non-dimensional deflection values of FGM plate are much lower than that of metal plate. This clearly shows that the FGM plate can very well resist high temperature conditions.

(c) As the value of volume fraction exponent 'n' is increased i.e. approaching towards metal rich region, the magnitude of deflection increases. For example in case of P-FGM at 400°C when n is equal to 0.5 the non-dimensional deflection is about 1, however when volume fraction exponent 'n' is increased to 10, the deflection is increased to 1.6. Further, for a constant value of exponent 'n', the non-dimensional deflection is found to vary linearly with rise in temperature.

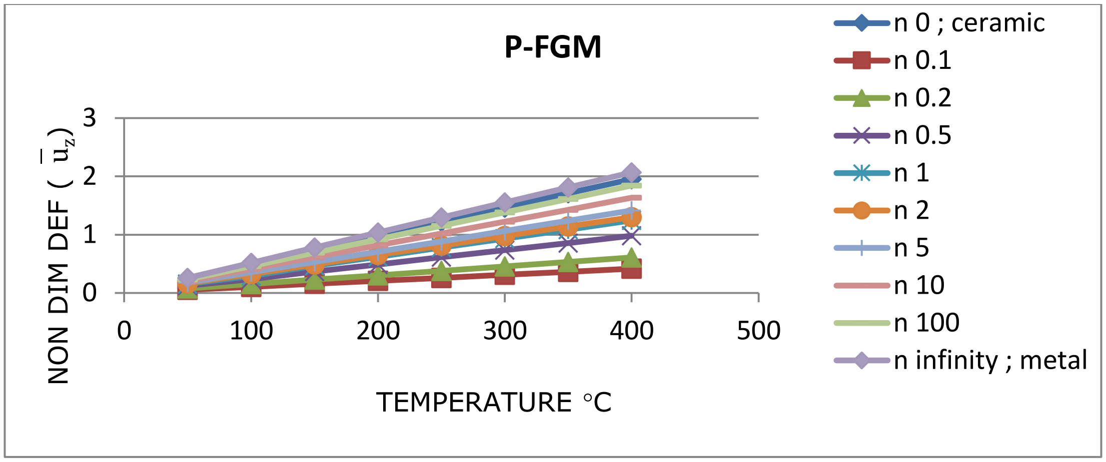

Figure 1: Effect of variable thermal environment on non-dimensional deflection ($\overline{u_z}$) for simply supported square plate (P-FGM)

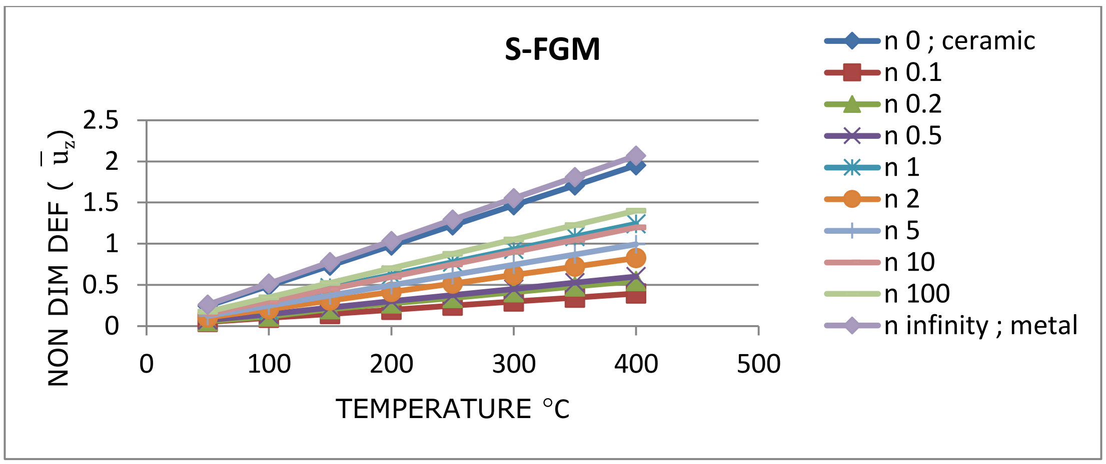

Figure 2: Effect of variable thermal environment on non-dimensional deflection ($\overline{u_z}$) for simply supported square plate (S-FGM)

(d) It is also found that at the low temperature range, for the various values of volume fraction exponent the deflections are closer to each other but as we increase the temperature, the deflection curve for the various volume fraction exponents are diverged.

b. Non-dimensional tensile stress ($\overline{\sigma_x}$)

The numerical results for variation of non-dimensional tensile stress ($\overline{\sigma_x}$) with temperature for simply supported plate in thermal environment for P-FGM and S-FGM are shown in Figure 3 and Figure 4 respectively. The comparison of results for various values of volume fraction exponent 'n' for P-FGM and S-FGM has been presented.

It can be observed from Figure 3 and Figure 4 that

(a) The non-dimensional tensile stress in the case of pure ceramic plate is comparable to that of pure metal plate. A similar trend is also observed in the analysis of FGM plate at constant thermal environment and the probable reason is already explained.

(b) As the value of volume fraction exponent 'n' is increased i.e. approaching towards metal rich region, the magnitude of non-dimensional tensile stress is increased. For example in case of P-FGM at 400°C when n is equal to 0.5 the non-dimensional tensile stress is about 120, however when volume fraction exponent ‘n’ is increased to 10, the tensile stress is increased to approximately 345.

(c) It is also found that in the low temperature region, for the various values of volume fraction exponent the non-dimensional tensile stress are closer to each other but as we increase the temperature, the non-dimensional tensile stress curves for the various volume fraction exponents are diverged. Further, the non-dimensional tensile stress is also found to vary linearly with rise in temperature.

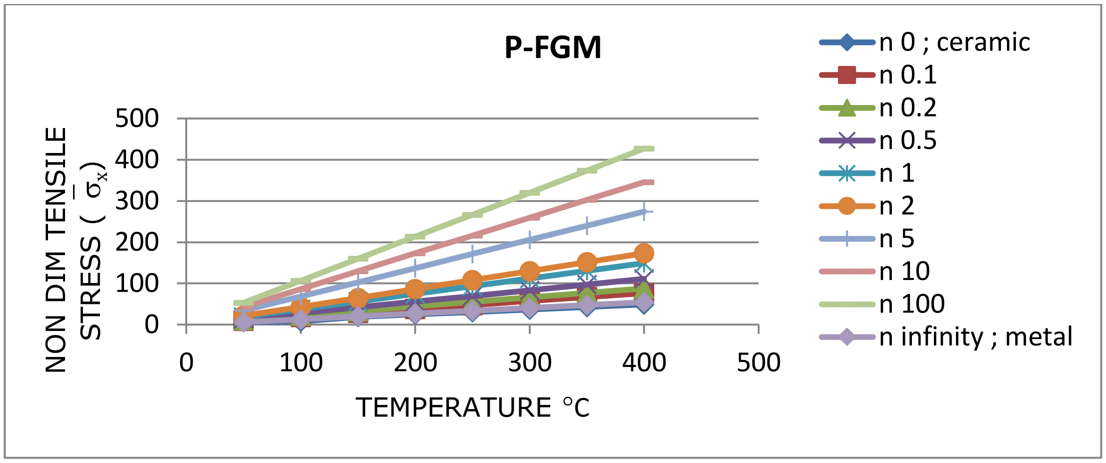

Figure 3: Effect of variable thermal environment on non-dimensional tensile stress ($\overline{\sigma_x}$) for simply supported square plate (P-FGM)

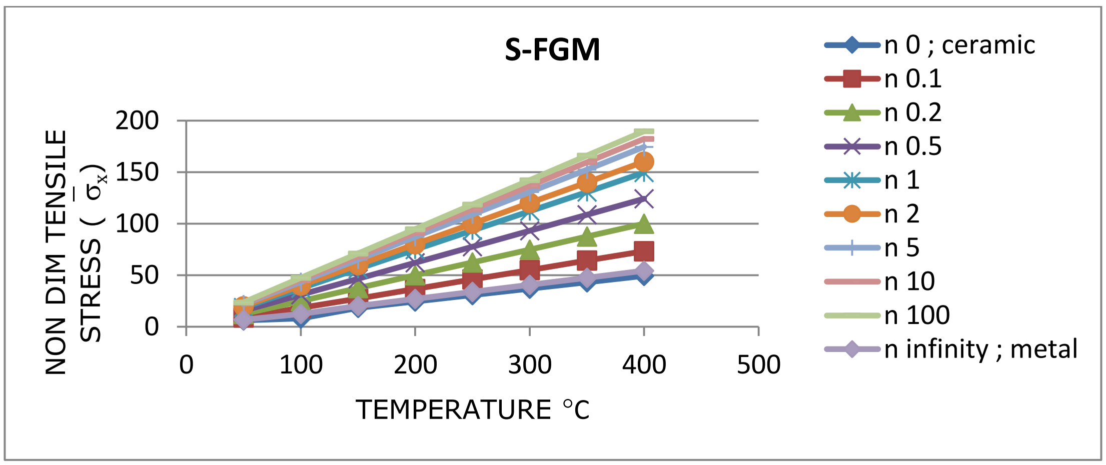

Figure 4: Effect of variable thermal environment on non-dimensional tensile stress ($\overline{\sigma_x}$) for simply supported square plate (S-FGM)

c. Non-dimensional shear stress ($\overline{\sigma_{xy}}$)

Figure 5 and Figure 6 show the effect of variation of temperature on non-dimensional shear stress ($\overline{\sigma_{xy}}$) for simply supported plate in thermal environment for P-FGM and S-FGM respectively. The comparison of results for various values of volume fraction exponent ‘n’ for P-FGM and S-FGM has been presented.

The following observations are made by comparing the non-dimensional shear stress ($\overline{\sigma_{xy}}$) for various values of temperature and types of FGM:

(a) The non-dimensional shear stress in the ceramic rich region and the metal rich region follow the trend similar to that has been observed in direct tensile stress as discussed in previous section (5.7.2).
(b) As the value of volume fraction exponent 'n' is increased i.e. approaching towards pure metal region, the magnitude of non-dimensional shear stress is increased. For example in case of P-FGM at 400°C when n is equal to 0.5 the non-dimensional shear stress is about 1718, however when volume fraction exponent 'n' is increased to 10, the shear stress is increased to approximately 1.5 times.
(c) It is also found that as the temperature is increased, the non-dimensional shear stress curves, for the various volume fraction exponents are diverged.

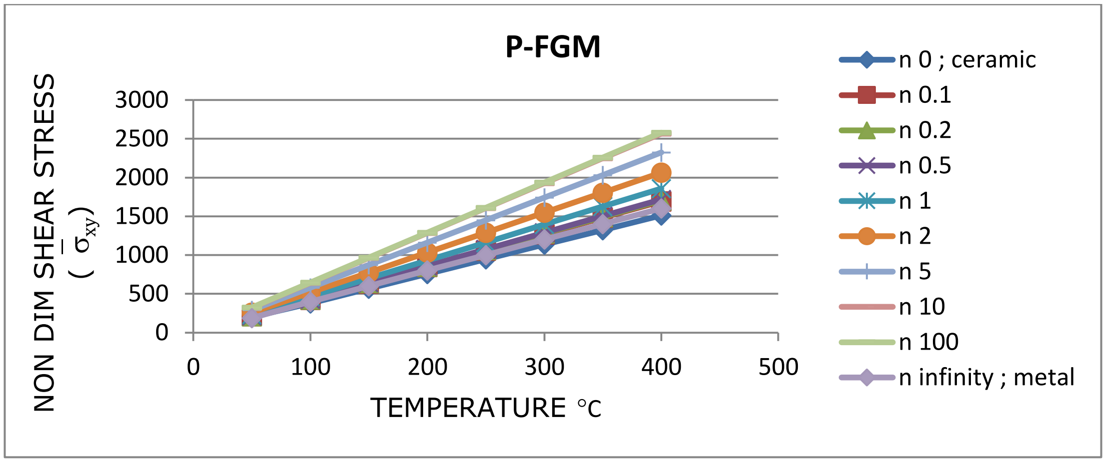

Figure 5: Effect of variable thermal environment on non-dimensional shear stress ($\overline{\sigma_{xy}}$) for simply supported square plate (P-FGM)

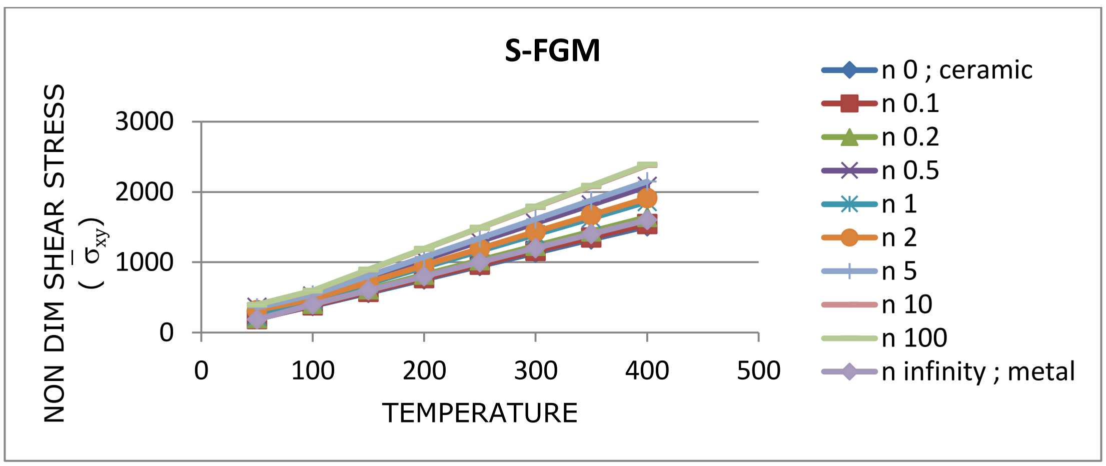

Figure 6: Effect of variable thermal environment on non-dimensional shear stress ($\overline{\sigma_{xy}}$) for simply supported square plate (S-FGM)

3.2 Variable thermal environment with constant mechanical load

In this section, the results of the analysis performed on a simply supported square (a/b=1) FGM plate subject to constant uniformly distributed load (1Xl0^6 N/m^2) with varying thermal environment are discussed. Thermo-mechanical analysis is performed by varying temperature of ceramic surface from 50°C to 400°C, while that of metallic surface, the temperature is kept 0°C. The effect of various volume fractions and various laws i.e. P-FGM and S-FGM are studied. The results are presented in terms of non-dimensional parameters i.e. non-dimensional deflection ($\overline{u_z}$), non-dimensional tensile stress ($\overline{\sigma_x}$) and non-dimensional shear stress ($\overline{\sigma_{xy}}$).

a. Non-dimensional deflection ($\overline{u_z}$)

Figure 7 and Figure 8 show the effect of variation of temperature on non-dimensional deflection ($\overline{u_z}$) temperature for simply supported plate under uniformly distributed load with variable thermal environment for P-FGM and S-FGM respectively. The comparison of results for various values of volume fraction exponent 'n' for P-FGM and S-FGM has been presented.

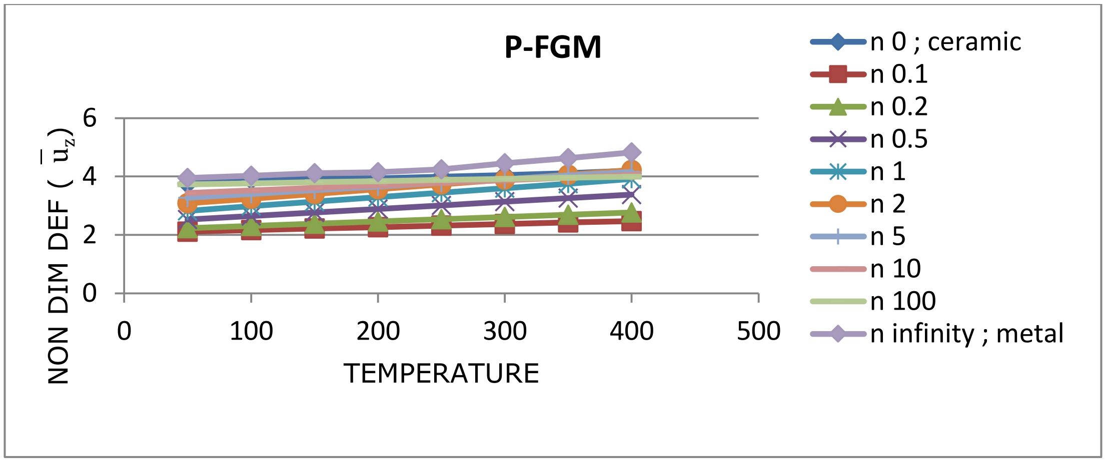

Figure 7: Effect of variable thermal environment on non-dimensional deflection ($\overline{u_z}$) for simply supported plate under constant mechanical load for P-FGM

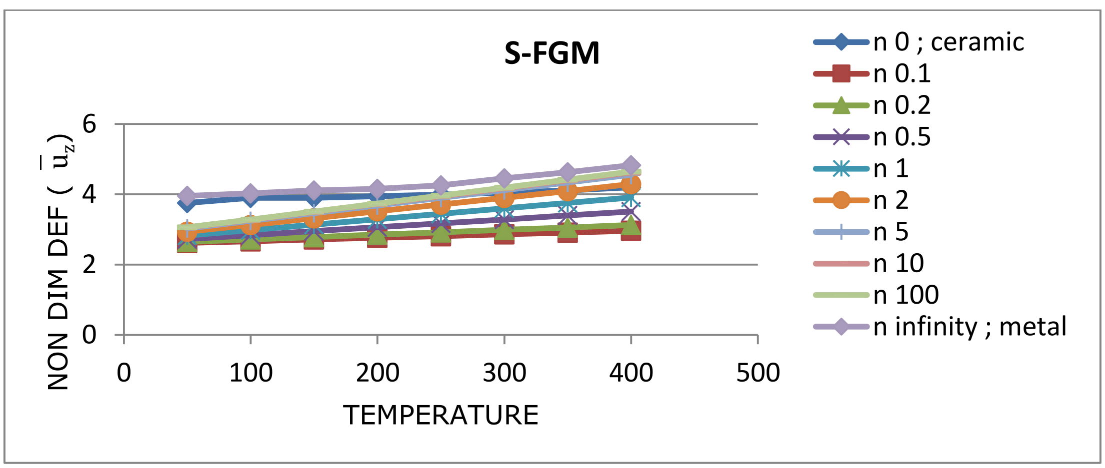

Figure 8: Effect of variable thermal environment on non-dimensional deflection ($\overline{u_z}$) for simply supported plate under constant mechanical load for S-FGM

A close study of the Figure 7 and Figure 8 reveals the following:
(a) The non-dimensional deflection increases linearly with temperature. Because of the high temperature at the top surface, the plate is having upward deflection. The deflection of the metal plate is maximum because the metal has high value of coefficient of thermal expansion.
(b) The deflection values of FGM plate are much lower than those of isotropic plate (e.g. fully ceramic or fully metal plate).
(c) As the value of volume fraction exponent 'n' is increased i.e. approaching towards pure metal, the magnitude of deflection increases. For example in case of P-FGM at 400°C when n is equal to 0.5 the non-dimensional deflection is about 3.4, however when volume fraction exponent 'n' is increased to 10, i.e. when metal content is increased, the deflection is increased to 4.5.
(d) Comparing the values of deflection under pure thermal and thermo-mechanical load, it is observed that the magnitude of deflection increases under thermo-mechanical load. For example in case of P-FGM at 400°C when n is equal to 0.5 the non-dimensional deflection is about 1 and that under thermo-mechanical load is 3.38. Also when volume fraction exponent 'n' is increased to 10, the deflection in case of P-FGM at 400°C is increased to 1.6 and that under thermo-mechanical load is 4.05.
(e) It is also found that at the lower temperature, for the various volume fraction exponents, the deflections are closer to each other but as we increase the temperature, the deflection for the various values of volume fraction exponent gets diverged.
(f) The non-dimensional deflection for S-FGM remains closer for various values of 'n' as compared to that of the P-FGM since material gradation is more uniform in S-FGM as compared to P-FGM.

b. Non-dimensional tensile stress ($\overline{\sigma_x}$)

The numerical results for variation of non-dimensional tensile stress ($\overline{\sigma_x}$) with temperature for simply supported plate under uniformly distributed load in variable thermal environment for P-FGM and S-FGM are shown in Figures 6.33 and 6.34 respectively. The comparison of results for various values of volume fraction exponent 'n' for P-FGM and S-FGM has been presented.
It can be observed from Figure 9 and Figure 10 that
(a) The non-dimensional tensile stress reduces steeply with increase in temperature for all the values of volume fractions.
(b) The plate is subjected to high temperature at the top surface and hence the plate deflects upward which is opposing the downward deflection due to the mechanical load. It shows that the stress due to temperature and stress due to mechanical load oppose each other and hence the tensile stress shows the reducing trend with increasing temperature under constant mechanical load.
(c) The non-dimensional tensile stress in the pure ceramic plate is comparable to that of pure metal plate. Probably this may be due to lower value of coefficient of thermal expansion and high value of modulus of elasticity for ceramic material.
(d) Comparing the values of non-dimensional tensile stress under pure thermal and thermo-mechanical load, it is observed that the magnitude of non-dimensional tensile stress increases under thermo-mechanical load. For example in case of P-FGM at 400°C when n is equal to 0.5 the non-dimensional tensile stress is about 111 and that under thermo-mechanical load is 138. Also when volume fraction exponent 'n' is increased to 10, the non-dimensional tensile stress in case of P-FGM 345 at 400°C is increased to 0 and that under thermo-mechanical load is 386.

(e) It is also found that at the lower temperature, for the various volume fraction exponents, the non-dimensional tensile stress are closer to each other but as we increase the temperature, the non-dimensional tensile stress for the various volume fraction exponent gets diverged.
(f) As the value of volume fraction exponent 'n' is increased i.e. approaching towards pure metal, the magnitude of non-dimensional tensile stress is increased. For example in case of P-FGM at 400°C when n is equal to 0.5 the non-dimensional tensile stress is about 139, however when volume fraction exponent 'n' is increased to 10, the tensile stress is increased to 386.

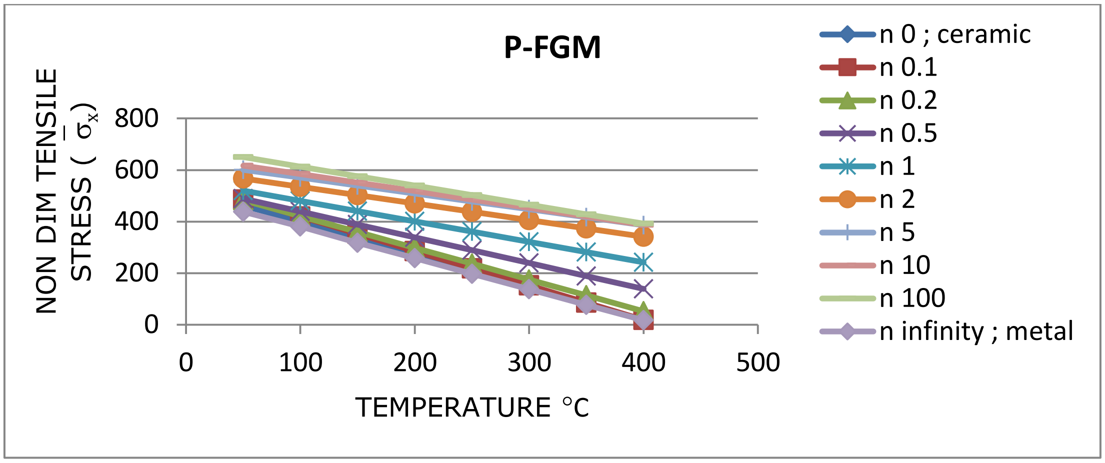

Figure 9: Effect of variable thermal environment on non-dimensional tensile stress ($\overline{\sigma_x}$) for simply supported plate under constant mechanical load for P-FGM

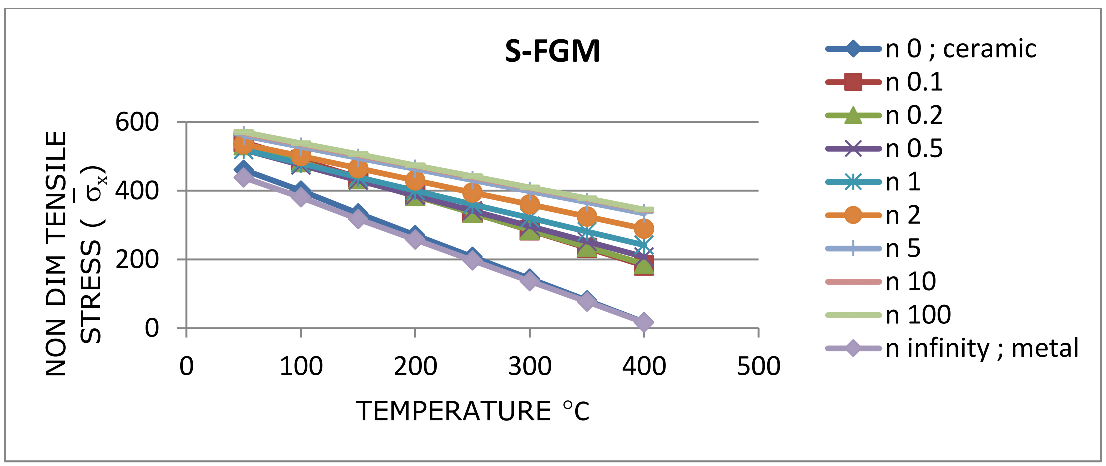

Figure 10: Effect of variable thermal environment on non-dimensional tensile stress ($\overline{\sigma_x}$) for simply supported plate under constant mechanical load for S-FGM

c. Non-dimensional shear stress ($\overline{\sigma_{xy}}$)

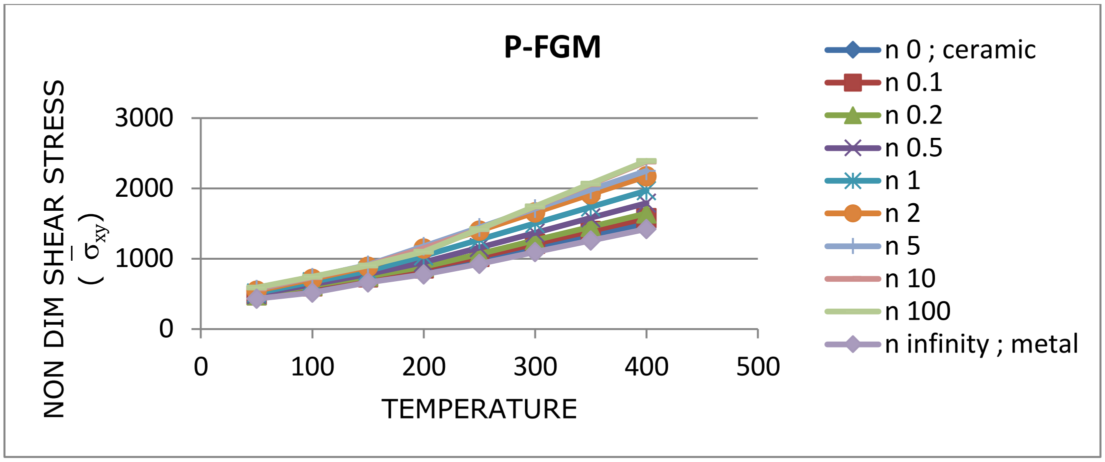

Figure 11: Effect of variable thermal environment on non-dimensional shear stress ($\overline{\sigma_{xy}}$) for simply supported plate under constant mechanical load for P-FGM

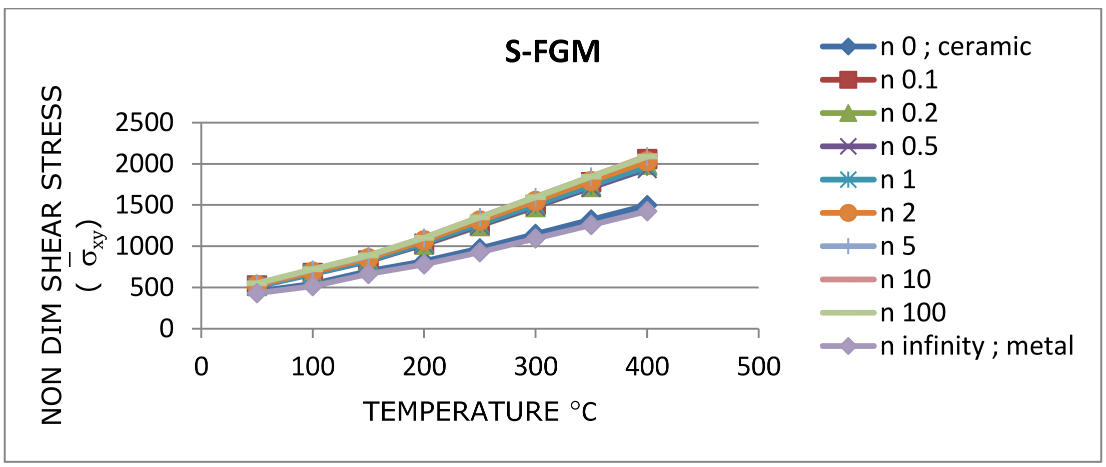

Figure 12: Effect of variable thermal environment on non-dimensional shear stress ($\overline{\sigma_{xy}}$) for simply supported plate under constant mechanical load for S-FGM

Figure 11 and Figure 12 show the effect of variation of temperature on non-dimensional shear stress ($\overline{\sigma_{xy}}$) for simply supported plate under uniformly distributed load in varying thermal environment for P-FGM and S-FGM respectively. The comparison of results for various values of volume fraction exponent 'n' for P-FGM and S-FGM has been presented.

The following observations are made by comparing the non-dimensional shear stress ($\overline{\sigma_{xy}}$) for various values of temperature and types of FGM:

(a) The non-dimensional shear stress in the ceramic plate and metal plate are comparable. Further the magnitude of the shear stress in either plate is found to be lower than FGM plates for various values of temperatures.

(b) As the value of volume fraction exponent 'n' is increased i.e. approaching towards pure metal, the magnitude of non-dimensional shear stress increased. For example in case of P-FGM at 400°C when n is equal to 0.5 the non-dimensional shear stress is about 1700, however when volume fraction exponent 'n' is increased to 10, the shear stress is increased to 2300.

(c) It is also found that at the lower temperature the non-dimensional shear stress are closer, for various values of 'n', to each other. However as the temperature is increased, the non-dimensional shear stress for the various volume fraction exponents diverge. A similar trend has been observed in variation of direct tensile stress. Further in case of S-FGM the effect of volume fraction exponent 'n' is found to be very small.

4. Conclusion

Thermal and thermomechanical responses of functionally graded ceramic-metal plates with varying thermal environment are analyzed. The non-dimensional deflection in the ceramic rich portion may be comparable to that in the metal rich region. It can be observed that the deflection values of FGM plates are much lower than those of isotropic plates (e.g. fully ceramic or fully metal plates) for a certain temperature range. The isotropic ceramic and metallic plate has the maximum deflection. This is clear that the FGM plates can resist high temperature conditions very well. It is also found that at the lower temperature rise for the various volume fraction exponents the deflections are closer to each other but as we increase the temperature, the deflection for the various volume fraction exponents diverge. It is also found that at some higher values of temperature for the some volume fraction exponents the deflections approaches and even increases beyond the value of deflection for the pure metal. The deflection therefore depends on the product of the temperature and the thermal expansion coefficient. It is observed that the isotropic ceramic metal plate has the lowest non dimensional tensile and shear stress. The non-dimensional tensile and shear stress becomes higher with increasing *n*. The response of the graded plates is not intermediate to the metal and ceramic plates. The non-dimensional tensile and shear stress reduces steeply for all the values of volume fractions. The non-dimensional deflection, tensile stress and shear stress for S-FGM remain closer for various values of 'n' as compared to that of the P-FGM. The work can be extended for variation in load, loading pattern, boundary conditions and other ceramic metal combinations.

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