NANO (1, 2)*LOCALLY CLOSED SETS IN NANO BITOPOLITICAL SPACES

S. Jeyashri*1, J. Arockia Jipsy*2

Assistant Professor, Department of Mathematics, Mother Teresa Women’s University, Kodaikanal
Research Scholar, Department of Mathematics, Mother Teresa Women’s University, Kodaikanal

*1jeyabalai@yahoo.com
*2arockiajipsy@gmail.com

Abstract—The notion of Nano topology was introduced by Lellis Thivagar etal. Bitopology was introduced and studied by J.C.Kelly. In this paper we introduced Nano(1, 2)*-locally closed sets in Nano bitopological spaces and also we discussed the concept of Nano(1, 2)*-LC-continuity in Nano bitopological spaces.

Keywords— Nano(1, 2)*-closed set, Nano(1, 2)*-locally closed set, Nano(1, 2)*-LC-continuity, Nano(1, 2)*-LC-irresolute

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I. INTRODUCTION

In 1963 Kelly [2] initiated the systematic study of bitopological spaces. 1989 Ganster and Reilly [1] studied locally closed sets in topological space. The notion of Nano topology was introduced by Lellis Thivagar [6,7]. In this paper we define Nano(1, 2)*-locally closed sets and study their properties in Nano bitopological spaces, and also we discussed the concept of Nano(1, 2)*-LC-continuity in Nano bitopological spaces.

II. PRELIMINARIES

Definition 2.1.,[6,7]
Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let X ⊆ U.
(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by \( L_R(X) \). That is,
\[
L_R(X) = \bigcap_{x \in X} \{ R(x) : R(x) \subseteq X \}
\]
where \( R(x) \) denotes the equivalence class determined by X.
(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by \( U_R(X) \). That is,
\[
U_R(X) = \bigcup_{x \in X} \{ R(x) : R(x) \cap X = \emptyset \}
\]
(iii) The boundary region of X with respect to R is the set of all objects, which can be neither in nor as not-X with respect to R and it is denoted by \( B_R(X) \). That is,
\[
B_R(X) = U_R(X) - L_R(X)
\]

Definition 2.2.,[6,7]
If (U, R) is an approximation space and X, Y ⊆ U, then
(i) \( L_R(X) \subseteq X \subseteq U_R(X) \)
(ii) \( L_R(\emptyset) = U_R(\emptyset) = \emptyset \)
(iii) \( U_R(X \cup Y) = U_R(X) \cup U_R(Y) \)
(iv) \( U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y) \)
(v) \( L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y) \)
(vi) \( L_R(X \cap Y) = L_R(X) \cap L_R(Y) \)
(vii) \( L_R(X) \subseteq L_R(Y) \) and \( U_R(X) \subseteq U_R(Y) \) whenever \( X \subseteq Y \)
(viii) \( U_R(X) = [L_R(X)]^C \) and \( L_R(X) = [U_R(X)]^C \)
(ix) \( U_R(X) = L_R(X) \) \( U_R(X) = U_R(X) \)
(x) \( L_R(X) = U_R(X) \) \( L_R(X) = L_R(X) \)

\( \tau_R(X) = \{ U, \phi, L_R(X), U_R(X), B_R(X) \} \)

**Definition 2.3** (6, 7)

Let \( U \) be an universe, \( R \) be an equivalence relation on \( U \) and \( \tau_R(X) = \{ U, \phi, L_R(X), U_R(X), B_R(X) \} \) where \( X \subseteq Y \). \( \tau_R(X) \) satisfies the following axioms
(i) \( U, \phi \in \tau_R(X) \)
(ii) The union of the elements of any sub-collection of \( \tau_R(X) \) is in \( \tau_R(X) \)
(iii) The intersection of the elements of any finite sub-collection of \( \tau_R(X) \) is in \( \tau_R(X) \).

That is, \( \tau_R(X) \) forms a topology on \( U \) called the nano topology on \( U \) with respect to \( X \). We call \( (U, \tau_R(X)) \) a nano topological space. The elements of \( \tau_R(X) \) are called nano open sets.

**Definition 2.4**

Let \( A \subseteq U \). Then \( A \) is called \( \tau_{R_{12}}(X) \)-open if \( A = A \cup B \), where \( A_i \in \tau_{R_{12}}(X) \) and \( B \in \tau_{R_{12}}(X) \). The complement of \( \tau_{R_{12}}(X) \)-open set is called \( \tau_{R_{12}}(X) \)-closed set. \( (U, \tau_{R_{12}}(X)) \) is called nano bitopological spaces.

**Definition 2.5**

If \( (U, \tau_{R_{12}}(X)) \) is a Nano bitopological spaces with respect to \( X \) where \( X \subseteq U \) and if \( A \subseteq U \), then
(i) The Nano \((1, 2)\)-interior of \( A \) is defined as the union of all Nano \((1, 2)\)-open subsets of \( A \) contained in \( A \) and it is denoted by \( \text{N}_1\text{int}_{(1,2)}(A) \). \( \text{N}_1\text{int}_{(1,2)}(A) \) is the largest Nano \((1, 2)\)-open subset of \( A \).
(ii) The Nano \((1, 2)\)-closure of \( A \) is defined as the intersection of all Nano \((1, 2)\)-closed sets containing \( A \) and it is denoted by \( \text{N}_1\text{cl}_{(1,2)}(A) \). \( \text{N}_1\text{cl}_{(1,2)}(A) \) is the smallest Nano \((1, 2)\)-Closed set containing \( A \).

**III. Nano \((1, 2)\)-Locally Closed Set**

**Definition 3.1**

A subset \( A \) of a Nano bitopological space \( (U, \tau_{R_{12}}(X)) \) is called Nano \((1, 2)\)-locally closed if \( A = L \cup M \) where \( L \in \text{Nano}(1, 2) \)-O(X) and \( M \in \text{Nano}(1, 2) \)-C(X), or equivalently if \( A = L \cap M \left( \text{N}_1\text{cl}_{(1,2)}(A) \right) \), for some \( L \in \text{Nano}(1, 2) \)-O(X).

**Note 3.2.**

Collection of all Nano \((1, 2)\)-locally closed set of \( (U, \tau_{R_{12}}(X)) \) is denoted by Nano \((1, 2)\)-LC(X).

**Example 3.3.**

Let \( U = \{ 1, 2, 3, 4 \} \) with \( U/R = \{ \{1\}, \{2, 3\}, \{4\} \} \). Let \( X_1 = \{ 1, 3 \} \subseteq U \). Then \( \tau_{R_{12}}(X_1) = \{ U, \phi, \{ 1 \}, \{ 2, 3 \}, \{ 1, 2, 3 \} \} \).
Let \( X_2 = \{ 2, 4 \} \subseteq U \). Then \( \tau_{R_{12}}(X_2) = \{ U, \phi, \{ 4 \}, \{ 2, 3, 4 \}, \{ 2, 3 \} \} \).

Then \( \text{Nano}(1, 2) \)-O(X) = \{ \{ U, \phi, \{ 1, 4 \}, \{ 1, 4 \}, \{ 2, 3 \}, \{ 1, 2, 3 \}, \{ 2, 3, 4 \} \} \) and \( \text{Nano}(1, 2) \)-C(X) = \{ \{ U, \phi, \{ 1, 4 \}, \{ 1, 4 \}, \{ 2, 3 \}, \{ 1, 2, 3 \}, \{ 2, 3, 4 \} \} \).

Hence Nano(1, 2)-LC(X) = \{ \{ U, \phi, \{ 1, 4 \}, \{ 1, 4 \}, \{ 2, 3 \}, \{ 1, 2, 3 \}, \{ 2, 3, 4 \} \} \}

**Remark 3.4.**

Every Nano(1, 2)-open set and Nano(1, 2)-closed set is Nano(1, 2)-locally closed, but the converse may not be true as seen in the following example.
Example 3.5.
Let $U = \{1, m, n, o\}$, $U/R := \{\{1\}, \{m, n\}, \{o\}\}$. 
Let $X_1 = \{m, n\} \subseteq U$. Then $\tau_{R_1}(X) = \{U, \phi, \{m, n\}\}$, $U/R := \{\{1, m\}, \{n\}, \{o\}\}$. 
Let $X_2 = \{1, n\} \subseteq U$. Then $\tau_{R_2}(X) = \{U, \phi, \{1\}, \{m, n\}\}$. 

Then 
Nano$(1, 2)^* - O(X) = \{U, \phi, \{n\}, \{1, m\}, \{m, n\}\}$ and 
Nano$(1, 2)^* - C(X) = \{U, \phi, \{o\}, \{n\}, \{1, m, o\}\}$. 

Now $\{1\}$ is neither $N_{\tau_{1,2}}$-open set nor a $N_{\tau_{1,2}}$-closed set even though $\{1\}$ is Nano$(1, 2)^*$-locally closed set as $\{1\} = \{1, m\} \cap \{1, o\}$.

Proposition 3.6.
Every nano $\tau_1$-open (resp., nano $\tau_1$-closed) set and nano$\tau_2$-open (resp., nano $\tau_2$-closed) sets is Nano$(1, 2)^*$-locally closed set.

Proof.
Since every nano$\tau_1$-open (resp., nano $\tau_1$-closed) set and nano $\tau_2$-open (resp., nano $\tau_2$-closed) set is Nano$(1, 2)^*$-open (resp., Nano$(1, 2)^*$-closed) set, hence they are Nano$(1, 2)^*$-locally closed set. We cannot that intersection of a Nano$(1, 2)^*$-locally closed set and nano$\tau_1$-open (or nano $\tau_1$-closed or nano $\tau_2$-open or nano $\tau_2$-closed) set is again a Nano$(1, 2)^*$-locally closed set.

Remark 3.7.
Converse of Proposition 3.6 may not be true in general as shown in the following example.

Example 3.8.
Let $U = \{p, q, r, s, t\}$, $U/R := \{\{p\}, \{q, r, s\}, \{t\}\}$. 
Let $X_1 = \{p, q\} \subseteq U$. Then $\tau_{R_1}(X) = \{U, \phi, \{p\}, \{q, r, s\}\}$. 
$U/R := \{\{p, q\}, \{r\}, \{s, t\}\}$. 
Let $X_2 = \{p, r\} \subseteq U$. Then $\tau_{R_2}(X) = \{U, \phi, \{p\}, \{r\}, \{q, s\}\}$. 

Then 
Nano$(1, 2)^* - O(X) = \{U, \phi, \{p\}, \{q, r\}, \{p, q\}\}$ and 
Nano$(1, 2)^* - C(X) = \{U, \phi, \{t\}, \{r\}, \{s, t\}\}$. 

Here $\{q, s, t\}$ is a Nano$(1, 2)^*$-locally closed set, but $\{q, s\}$ is not a $N_{\tau_1}$-open (resp., $N_{\tau_2}$-closed) set. Again $\{r, s, t\}$ is a Nano$(1, 2)^*$-locally closed set, but $\{r, s\}$ is not a $N_{\tau_2}$-open (resp., $N_{\tau_2}$-closed) set.

Theorem 3.9.
For a subset $A$ of $(U, \tau_{R_{1,2}}(X))$ the following conditions are equivalent:
(i) $A \in$ Nano$(1, 2)^*$-LC$(X)$, 
(ii) $A = P \cap N_{\tau_{1,2}}cl(A)$ for some $N_{\tau_{1,2}}$-open set $P$, 
(iii) $N_{\tau_{1,2}}cl(A) - A = N_{\tau_{1,2}}$-closed, 
(iv) $A \cup (U - N_{\tau_{1,2}}cl(A))$ is $N_{\tau_{1,2}}$-open.

Proof.
(i) $\Rightarrow$ (ii). Let $A \in$ Nano$(1, 2)^*$-LC$(X)$. Then there exist a $N_{\tau_{1,2}}$-open set $P$ and a $N_{\tau_{1,2}}$-closed set $F$ of $(U, \tau_{R_{1,2}}(X))$ such that $A = P \cap F$. Clearly $A \subseteq P \cap N_{\tau_{1,2}}cl(A)$. Since $N_{\tau_{1,2}}cl(A) \subseteq F$, $P \cap N_{\tau_{1,2}}cl(A) \subseteq P \cap F = A$. Thus $A = P \cap N_{\tau_{1,2}}cl(A)$.

(ii)$\Rightarrow$(iii). Let $F = N_{\tau_{1,2}}cl(A) - A$ Now, $U - F = U - \{N_{\tau_{1,2}}cl(A) - A\} = (A \cup F) - \{N_{\tau_{1,2}}cl(A) - A\} = (A \cup F) - \{N_{\tau_{1,2}}cl(A)\}$. Where, $(N_{\tau_{1,2}}cl(A)) \cap (U - N_{\tau_{1,2}}cl(A)) = \emptyset$. Where, $(U - N_{\tau_{1,2}}cl(A)) \cap (N_{\tau_{1,2}}cl(A)) = \emptyset$. Where, $(U - N_{\tau_{1,2}}cl(A)) \cap (N_{\tau_{1,2}}cl(A)) = \emptyset$.

(iii)$\Rightarrow$(iv). Since $F$ is $N_{\tau_{1,2}}$-closed, we get $U - F = U - \{N_{\tau_{1,2}}cl(A) - A\} = U \cap (N_{\tau_{1,2}}cl(A) - A) = (U \cap N_{\tau_{1,2}}cl(A))$. Therefore, $A \cup (U - N_{\tau_{1,2}}cl(A))$ is $N_{\tau_{1,2}}$-open.

(iv) $\Rightarrow$ (i). Since $A \cup (U - N_{\tau_{1,2}}cl(A))$ is $N_{\tau_{1,2}}$-open, this implies $A \cup (N_{\tau_{1,2}}cl(A))$ is $N_{\tau_{1,2}}$-open set. Thus $A$ is $N_{\tau_{1,2}}$-open. Hence is Nano$(1, 2)^*$-locally closed set (Since every
If open set) is called an infra bitopology on $N_{T_{1,2}}$-closed set.

**Theorem 3.10**

A subset $A$ of $(U, \tau_{T_{1,2}}(X))$ is Nano$(1, 2)^*$-locally closed if and only if $A^c$ is the union of a $N_{T_{1,2}}$-open set and a $N_{T_{1,2}}$-closed set.

**Proof.**

Proof follows from the definition.

**Remark 3.11.**

The union of any two Nano$(1, 2)^*$-locally closed sets may not be a Nano$(1, 2)^*$-locally closed set as shown in the following example.

**Example 3.12.**

$U = \{1, 2, 3, 4\}$, with $U/R = \{\{1\}, \{2, 4\}, \{3\}\}$ and $X_1 = \{1, 2\}$

Then $\tau_{R_{1,2}}(X) = \{U, \phi, \{1\}, \{1, 2, 4\}, \{2, 4\}\}$.

Let $U/R_2 = \{\{1\}, \{2, 3\}, \{4\}\}$, $X_2 = \{1, 3\}$ then

$\tau_{R_2}(X) = \{U, \phi, \{1\}, \{2, 3\}, \{1, 2, 3\}\}$.

Nano$(1, 2)^*$-O($X$)$= \{U, \phi, \{1\}, \{2, 3\}, \{2, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{2, 3, 4\}\}$

Nano$(1, 2)^*$-G($X$)$= \{U, \phi, \{1\}, \{2, 3\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{1, 2, 3\}, \{1, 2, 4\}, \{2, 3, 4\}\}$

Nano$(1, 2)^*$-LC($X$)$= \{U, \phi, \{1\}, \{2, 3\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{1, 2, 3\}, \{1, 2, 4\}, \{2, 3, 4\}\}$

Here $\{3\}$ and $\{4\}$ are two $(1, 2)^*$-locally closed sets, but $\{3, 4\}$ is not a $(1, 2)^*$-locally closed set.

**Definition 3.13.**

Any two subsets $A$ and $B$ in a Nano bitopological space $U$ are said to be Nano$(1, 2)^*$-separated sets if $A \cap N_{T_{1,2}}\text{cl}(B) = \phi = N_{T_{1,2}}\text{cl}(A) \cap B$.

**Proposition 3.14.**

Let $A$ and $B$ be any two subsets of $(U, \tau_{T_{1,2}}(X))$. Suppose that the collection of all Nano$(1, 2)^*$-open sets is closed under finite intersection. Let $A, B \in \text{Nano}(1,2)^*$-LC($X$), and if $A$ and $B$ are Nano$(1, 2)^*$-separated set in $(U, \tau_{T_{1,2}}(X))$, then $A \cup B \in \text{Nano}(1,2)^*$-LC($X$).

**Proof.**

By Theorem 3.9 there exist $N_{T_{1,2}}$-open sets $G$ and $H$ of $(U, \tau_{T_{1,2}}(X))$ such that $A = G \cap N_{T_{1,2}}\text{cl}(A)$ and $B = H \cap N_{T_{1,2}}\text{cl}(B)$, and $U = (G \cap \text{cl}(A)) \cap (H \cap \text{cl}(B)) = \phi$. Consequently $A \cup B = (U \cap N_{T_{1,2}}\text{cl}(A))^c = \phi$. Consequently $A \cup B \in \text{Nano}(1,2)^*$-LC($X$).

**Proposition 3.15.**

In any Nano bitopological space $(U, \tau_{T_{1,2}}(X))$ intersection of any two Nano$(1, 2)^*$-locally closed is a Nano$(1, 2)^*$-locally closed set.

**Proof.**

The proof is straightforward. Studying the properties of Nano$(1, 2)^*$-locally closed set we have the idea to define a new concept in a Nano bitopological space $(U, \tau_{T_{1,2}}(X))$, namely, infra bitopology, and we also introduce Nano$(1, 2)^*$-LC-continuity using Nano$(1, 2)^*$-locally closed sets.

**Definition 3.16.**

Let $(U, \tau_{T_{1,2}}(X))$ be a Nano bitopological space. Any collection $NT_{1,2}$ of subsets of $U$ (strongly related with $N_{T_{1,2}}$-open set) is called an infra bitopology on $U$.

If

(i) $\phi, U \in N_{T_{1,2}}$;

(ii) $NT_{1,2}$ is closed under finite intersection. Then the space $(U, T_{1,2})$ is called an infra nano bitopological space.
Theorem 3.17.
Let \((U, \tau_{R_{1,2}}(X))\) be a Nano bitopological space. Then the collections of all Nano(1, 2)*-locally closed sets of \((U, \tau_{R_{1,2}}(X))\) denoted by Nano(1, 2)*-LC(X) are forms an infra Nano bitopology on U.

Proof.: 
Proposition 3.15 serves the purpose.

Definition 3.18.
The infra Nano bitopology defined above is called Nano(1, 2)*-LC infra Nano bitopology and the space (U, Nano(1, 2)*-LC(U)) is called NLC-infra Nano bitopological space.

Example 3.19.
Let \(U = \{p, q, r, s, t\}\). Let \(X = \{p, q, r, s, t\}\). Let \(\tau_{R_{1,2}}(X) = \{U, \phi, \{p\}, \{q, r, s\}, \{q, r, s, t\}\}\). Then Nano(1, 2)*-O(X) = \(\{U, \phi, \{p\}, \{q, r, s\}, \{q, r, s, t\}\}\) and Nano(1, 2)*-C(X) = \(\{U, \phi, \{t\}, \{q, r, s\}, \{q, r, s, t\}\}\).

NANO(1, 2)*-LC(X) = \(\{U, \phi, \{p, r\}, \{q, r, s\}, \{q, r, s, t\}\}\) is called Nano(1, 2)*-closed set if and only if \(N\tau_{1,2}^{*} \cap \tau_{1,2}^{*} = \phi\).

Theorem 3.20.
If \(A \in \text{Nano}(1, 2)^{*}\)-LC(X) and \(B \in \text{Nano}(1, 2)^{*}\)-C(X), then \(A \cap B \in \text{Nano}(1, 2)^{*}\)-LC(X).

Proof.: 
It is obvious because every Nano(1, 2)*-closed set is Nano(1, 2)*-locally closed set and intersection of two Nano(1, 2)*-locally closed sets is also a Nano(1, 2)*-locally closed.

Definition 3.21.
A subset \(A\) of \((U, \tau_{R_{1,2}}(X))\) is called Nano(1, 2)*-dense(resp., Nano(1, 2)*-nowhere dense) set if and only if \(N\tau_{1,2}^{*} \cap \tau_{1,2}^{*} = \phi\).

Theorem 3.22.
Any Nano(1, 2)*-dense subset of \((U, \tau_{R_{1,2}}(X))\) is Nano(1, 2)*-locally closed if and only if it is \(N\tau_{1,2}^{*}\)-open.

Proof.
Let us assume that \(A\) be any Nano(1, 2)*-dense set in \((U, \tau_{R_{1,2}}(X))\). Now by the given hypothesis \(A = U \cap N\tau_{1,2}^{*}\)-closed set for some \(U \in \text{Nano}(1, 2)^{*}\)-O(X), this implies that \(A\) is a \(N\tau_{1,2}^{*}\)-open set. Conversely let \(A\) be any \(N\tau_{1,2}^{*}\)-open set in \((U, \tau_{R_{1,2}}(X))\). Now since every \(N\tau_{1,2}^{*}\)-open set is Nano(1, 2)*-locally closed set, therefore \(A\) is Nano(1, 2)*-locally closed set.

Theorem 3.23.
If any Nano(1, 2)*-dense subset of \(U\) is \(N\tau_{1,2}^{*}\)-open, then every subset \(A\) of \(U\) is Nano(1, 2)*-locally closed.

Proof.
Let \(B_{1}\) be any Nano(1, 2)*-dense subset in \(U\). Therefore \(B_{1} = A \cup (N\tau_{1,2}^{*}\)-closed set). Again let \(B_{2}\) be the \(N\tau_{1,2}^{*}\)-closed set; that is, \(B_{2} = A \cup (N\tau_{1,2}^{*}\)-closed set). Let \(B_{1} \cap B_{2} = A \cup (N\tau_{1,2}^{*}\)-closed set) \(\cap A \cup (N\tau_{1,2}^{*}\)-closed set) = A\). This implies that \(A\) is Nano(1, 2)*-locally closed set.

IV. Nano (1,2)* - LC CONTINUOUS AND Nano (1,2)* - LC IRRESOLUTE
Throughout this paper U, V, and W denote the Nano bitopological spaces \((U, \tau_{R_1}, \tau_{R_2})\), \((V, \sigma_{R_1}, \sigma_{R_2})\), and \((W, \eta_{R_1}, \eta_{R_2})\), respectively, on which no separation axioms are assumed. The concept of Nano(1, 2)*-continuous function from a Nano bitopological space U into another Nano bitopological space V was defined as follows.

**Definition 4.1.**
A function \(f: (U, \tau_{R_1}, \tau_{R_2}) \rightarrow (V, \sigma_{R_1}, \sigma_{R_2})\) is said to be Nano(1, 2)*-continuous if and only if inverse image of every \(N\sigma_{1,2}\)-openset in \(V\) is \(N\tau_{1,2}\)-open set in U.

**Definition 4.2.**
A mapping \(f: (U, \tau_{R_1}, \tau_{R_2}) \rightarrow (V, \sigma_{R_1}, \sigma_{R_2})\) is said to be Nano(1, 2)*-LC continuous if and only if the inverse image of every \(N\sigma_{1,2}\)-open set in \(V\) is Nano(1, 2)*-locally closed in U.

**Proposition 4.3.**
If \(f: (U, \tau_{R_1}, \tau_{R_2}) \rightarrow (V, \sigma_{R_1}, \sigma_{R_2})\) is Nano(1, 2)*-LC continuous and \(g: (V, \sigma_{R_1}, \sigma_{R_2}) \rightarrow (W, \eta_{R_1}, \eta_{R_2})\) is Nano(1, 2)*-continuous, then \((g \circ f)\) is Nano(1, 2)*-LC continuous.

**Proof:**
Proof is obvious.

**Theorem 4.4.**
A function \(f: (U, \tau_{R_1}, \tau_{R_2}) \rightarrow (V, \sigma_{R_1}, \sigma_{R_2})\) is Nano(1, 2)*-continuous if it is Nano(1, 2)*-LC continuous and the preimage of every \(N\sigma_{1,2}\)-open set is Nano(1, 2)*-dense in U.

**Proof:**
Let \(A\) be any Nano(1, 2)*-open set in \(V\). Since \(f\) is Nano(1, 2)*-LC continuous, then \(f^{-1}(A)\) is Nano(1, 2)*-locally closed in U. By hypothesis \(N\tau_{1,2}\)-cl\((f^{-1}(A)) = U\). Then \(f^{-1}(A)\) is \(N\tau_{1,2}\)-open in U. Hence \(f\) is Nano(1, 2)*-continuous (by Theorem 3.14).

**Definition 4.5.**
A function \(f: (U, \tau_{R_1}, \tau_{R_2}) \rightarrow (V, \sigma_{R_1}, \sigma_{R_2})\) is called Nano(1, 2)*-LC-irresolute if and only if inverse image of every Nano(1, 2)*-locally closed set in \(V\) is Nano(1, 2)*-locally closed set in U.

**Theorem 4.6.**
Let \(f: (U, \tau_{R_1}, \tau_{R_2}) \rightarrow (V, \sigma_{R_1}, \sigma_{R_2})\) and \(g: (V, \sigma_{R_1}, \sigma_{R_2}) \rightarrow (W, \eta_{R_1}, \eta_{R_2})\) be any two functions; then

(i) \((g \circ f)\) is Nano(1, 2)*-LC-irresolute if \(g\) is Nano(1, 2)*-LC-irresolute and \(f\) is Nano(1, 2)*-LC-irresolute;

(ii) \((g \circ f)\) is Nano(1, 2)*-LC continuous if \(g\) is Nano(1, 2)*-LC continuous and \(f\) is Nano(1, 2)*-LC-irresolute.

**Proof.**
(i) Let \(V\) be Nano(1, 2)*-locally closed set in \((W, \eta_{R_1}, \eta_{R_2})\). Since \(g\) is Nano(1, 2)*-LC-irresolute, then \(g^{-1}(V)\) is Nano(1, 2)*-locally closed set in \((V, \sigma_{R_1}, \sigma_{R_2})\). As \(f\) is Nano(1, 2)*-LC-irresolute so \(f^{-1}(g^{-1}(V))\) is Nano(1, 2)*-locally closed set in \((U, \tau_{R_1}, \tau_{R_2})\). Hence \((g \circ f)\) is Nano(1, 2)*-LC-irresolute.

(ii) Let \(V\) be \(\eta_{1,2}\)-open set in \((W, \eta_{R_1}, \eta_{R_2})\). Since \(g\) is Nano(1, 2)*-LC-continuous, so \(g^{-1}(V)\) is Nano(1, 2)*-locally closed set in \((V, \sigma_{R_1}, \sigma_{R_2})\). Also \(f\) is \((1, 2)^*\)-LC-irresolute so \(f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)\) is Nano(1, 2)*-locally closed set in \((U, \tau_{R_1}, \tau_{R_2})\). Therefore \((g \circ f)\) is Nano(1, 2)*-LC-continuous.

**Example 4.7**
\(U = \{1, 2, 3, 4\}\), with \(U/R_1 = \{\{1\}, \{2, 4\}, \{3\}\}\) and \(X_1 = \{1, 2\}\).

Then \(\tau_{R_2}(X) = \{U, \emptyset, \{1\}, \{1, 2, 4\}, \{2, 4\}\}\).
Let $U/R_2 = \{\{1\},\{2,3\},\{4\}\}$, $X_2 = \{1,3\}$ then $\tau_{R_2}(X) = \{U, \phi, \{1\}, \{2,3\}, \{1,2,3\}\}$.

Nano$(1, 2)^*-O(X) = \{U, \phi, \{1\}, \{2,3\},\{4\}\}$, $\tau_{R_2}(X) = \{U, \phi, \{1\}, \{2,3\},\{4\}\}$

Nano$(1,2)^*-LC(X) = \{U, \phi, \{1\}, \{3\}, \{4\}\}$, $\tau_{R_2}(X) = \{U, \phi, \{1\}, \{3\}, \{4\}\}$

Let $V = \{a, b, c, d\}$, with $V/R_1 = \{\{a\}, \{c\}, \{b, d\}\}$ and $X_1 = \{a, b\}$ then the nano topology,

$\sigma_{R_2}(X) = \{V, \phi, \{a\}, \{b, d\}\}$. with $V/R_2 = \{\{a\}, \{b\}\}$ and $X_2 = \{a, d\}$

Define: $f: (U, \tau_{R_2}, \tau_{R_2}) \rightarrow (V, \sigma_{R_2}, \sigma_{R_2})$ as an identity map. Then $f$ is a Nano$(1, 2)^*-LC$-continuity and $f$ is a Nano$(1, 2)^*-LC$ irresolute.

V. CONCLUSIONS

Many different forms of locally closed set have been introduced over the years. Various interesting problems arise when one considers openness. Its importance is significant in various areas of mathematics and related sciences, in this paper we studied the concept of Nano$(1, 2)^*-ss$-locally closed set in Nano bitopological spaces. Also it is proved that every Nano$(1, 2)^*-locally closed set is Nano$(1, 2)^*-closed set if it is Nano$(1, 2)^*-nowhere dense set in U$. There is a scope to study the concept of strongly and perfectly continuous mapping in a Nano bitopological space and interrelationship between stronger form and weaker form of Nano$(1, 2)^*-continuity and Nano(1, 2)^*-LC$-continuous mapping.

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