Discrete symmetries in dyonic fields

Ila joshi[1] and Jivan Singh Garia[2]
[1]Research Scholar, Department of Physics, L. S. M. Govt. P.G. College, Pithoragarh,U. K.
[2]Assistant Professor, Department of Physics, L. S. M. Govt. P. G. College, Pithoragarh, U.K.

Abstract
Dyon is a hypothetical particle which carries the electric and magnetic charge. We have made an attempt to discuss the role of discrete symmetries in the dyonic field. We report the symmetry transformations under the parity (P), time reversal (T), charge conjugation (C), time-reversal (PT), Charge-conjugation and parity (CP) and CPT to various equations of dyon. These symmetries are also obtained in term of polarization and magnetization vector for dyon. It is shown that CPT transformation is invariant under electric and magnetic charge polarization and magnetization vector of dyon respectively.
Keywords: *Dyon, Electromagnetic fields, Polarization, Magnetization*

1. Introduction

Discrete symmetries do not depend on any continuous parameter. The classic example is reflection in a mirror-either we reflect an object in the mirror, or we leave it unchanged. The study of interactions between the particles has led to some conservation laws which govern the symmetry in the model [1–4]. Three discrete symmetries play crucial roles in particle physics: parity (P), charge conjugation (C), and time-reversal (T). We have discussed the generalized Dirac–Maxwell (GDM) equations in the presence of electric and magnetic sources. We have also analyzed the other quantum equations of dyon inconsistent and manifest covariant way [5]. This theory has been shown to remain invariant under the duality transformation. Quaternion analysis of time-dependent Maxwell's equations has been developed [6] in the presence of electric and magnetic charges and the solution for the classical problem of moving charge (electric and magnetic) are obtained consistently. The time-dependent generalized Dirac–Maxwell's (GDM) equations of dyon have also been discussed [7] in inhomogeneous and chiral media, where the solutions for the classical problem are obtained. The monochromatic fields of generalized electromagnetic fields of dyons in slowly changing media has been derived in a consistent manner[14].

In this paper, we have attempted to study the different symmetric properties of dyon in term of polarization and magnetization. It is shown that the generalized Maxwell's–Dirac (GDM) equations are invariant under discrete symmetries like parity P, time reversal T, charge conjugation (C), parity–time reversal operation PT, charge conjugation–parity operation CP and combined operation (CPT). The generalized field equations are no more invariant under P symmetry, T transformations and combined effects of CP and PT. As such, CPT invariance is an exact symmetry for generalized fields of dyon [8]. If we apply these transformations to field associated with dyon, we conclude that CPT transformation is also invariant for both polarization and magnetization vector terms of dyon.

Field Associated with Dyon

Considering the existence of magnetic monopoles Dirac [9] generalized Maxwell's field equation for dyon in the following manner in a vacuum in SI units [10] for $c = \eta = 1$

$$\vec{\nabla}.\vec{E} = \frac{\rho_e}{\varepsilon_0} \tag{1}$$

$$\vec{\nabla}.\vec{B} = \mu_0 \rho_m \tag{2}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \frac{\vec{j}_m}{\varepsilon_0} \tag{3}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}_e + \frac{\partial \vec{E}}{\partial t} \tag{4}$$

Generalized electromagnetic fields produced by particle (dyon) each carrying the generalized charge as complex quantity with electric and magnetic charges as its real and imaginary constituents. The electric and magnetic field associated with dyon carrying generalized charge may be written as [11-12].

$$q = e - i\,g$$

Electric and magnetic charges are represented by e and g, respectively. Generalized four potential $\{V_\mu\} = (\phi, \vec{V})$ associated with dyon is defined as,

$$\{V_\mu\} = \{C_\mu\} - i\{D_\mu\} \tag{5}$$

Where $\{C_\mu\} = (\phi_e, \vec{C})$ and $\{D_\mu\} = (\phi_g, \vec{D})$ are respectively electric and magnetic four potentials in term of scalar and vector part. We have used the natural units $c = \eta = 1$. Electric and magnetic fields of dyon are defined in terms of components of electric and magnetic potentials as

$$\vec{E} = -\vec{\nabla}\phi_e - \frac{\partial \vec{C}}{\partial t} - \vec{\nabla} \times \vec{C} \tag{6}$$

$$\vec{B} = -\vec{\nabla}\phi_m - \frac{\partial \vec{D}}{\partial t} + \vec{\nabla} \times \vec{D} \tag{7}$$

The vector wave function $\vec{\Psi}$ associated with generalized electromagnetic fields is defined as,

$$\vec{\psi} = \vec{E} - i\vec{B} \tag{8}$$

We get following the form of generalized Maxwell's equations for dyon.

$$\vec{\nabla}.\vec{\psi} = \rho \tag{9}$$

$$\vec{\nabla} \times \vec{\Psi} = -i\vec{J} - i\frac{\partial \vec{\Psi}}{\partial t} \tag{10}$$

Where ρ and J, are the generalized charge and current source densities of dyon and P and M, are polarization and magnetization vector fields given by

$$\rho = \rho_e - i\rho_m \tag{11}$$

$$\vec{J} = \vec{j}_e - i\vec{j}_m \tag{12}$$

The Generalized covariant form of Maxwell equations of dyon are written as.

$$F_{\mu\nu,\nu} = j^e_\mu \qquad F^d_{\mu\nu)} = j^m_\mu \tag{13}$$

The generalized field equation of dyon is given by

$$G_{\mu\nu,\nu} = J_\mu \tag{14}$$

Where

$$J_\mu = (\rho, -J) \tag{15}$$

Symmetries of Dyon under parity transformation

Symmetries under parity are essential in nuclear and particle physics and many other areas of quantum physics. It describes the effect on a system of reversing all coordinates: x→-x, y →-y, and z →- z, since this is often the case for systems of interest, the parity transformation referred to as "reflection in a mirror." Parity is the symmetry of interaction which involves a transformation that changes the algebraic sign of the coordinate system [1, 2]. Right-handed coordinates are changed into a left-handed coordinate one or vice versa under the parity transformation. The physical quantities like electric charge density (ρ_e), magnetic current density, Polarization vector for electric charge and magnetic field have even parity as they do not change [13] their sign under spatial inversion (Parity transformation) showing that they have even parity.

$$P(\rho_e)P^{-1} = \rho_e, \quad P(\vec{j}_m)P^{-1} = \vec{j}_m, \quad P(\vec{B})P^{-1} = \vec{B}, \quad P(\vec{M}_e)P^{-1} = \vec{M}_e, \quad P(\vec{P}_m)P^{-1} = \vec{P}_m \tag{16}$$

On the other hand, the physical quantities like magnetic charge density (ρ_m), electric current density, electric field (E), Polarization vector for electric charge, and magnetization vector for magnetic charge have odd parity as they change sign [13] their sign change under parity transformation.

(8)

$$P(\vec{j}_e)P^{-1} = -\vec{j}_e, \quad P(\rho_m)P^{-1} = -\rho_m, \quad P(\vec{P}_e)P^{-1} = -\vec{P}_e, \quad P(\vec{E})P^{-1} = -\vec{E}, \quad P(\vec{M}_m)P^{-1} = -\vec{M}_m \tag{17}$$

On applying the parity transformation to field associated with dyon. We get

$$P(\vec{\psi})P^{-1} = -\vec{\psi}, \quad P(\vec{V})P^{-1} = -\vec{V}, \quad P(\vec{\rho})P^{-1} = \rho, \quad P(\vec{J})P^{-1} = -\vec{J}, \quad P(\vec{P})P^{-1} = -\vec{P}, \quad P(\vec{M})P^{-1} = \vec{M} \tag{18}$$

(11)

(12)

As such, the covariant form of Maxwell equations of dyon is invariant under parity transformation. But Polarization vector for dyon looses their invariance under time reversal transformation.

Symmetries of Dyon under Time reversal transformation

The final discrete symmetry is time reversal, T, which reverses the flow of time: t →-t. Thus the physical quantities like electric charge density (ρ_e), magnetic current density, Polarization vector for electric charge, and magnetic field have even parity as they do not change [13] their sign under time reversal. i. e.

$$T(x)T^{-1} = x, \quad T(\rho_e)T^{-1} = \rho_e, \quad T(\vec{j}_m)T^{-1} = \vec{j}_m, \quad T(\Phi_e)T^{-1} = \Phi_e, \quad T(\vec{P}_e)T^{-1} = \vec{P}_e, \quad T(\vec{M}_m)T^{-1} = \vec{M}_m \tag{19}$$

On the other hand, the physical quantities like electric charge density, magnetic field, polarization vector for the magnetic charge and Magnetization vector for the electric charge are changing their sign under time reversal i.e

$$T(t)T^{-1} = -t, \quad T(\vec{P}_m)T^{-1} = -\vec{P}_m, \quad T(\vec{M}_e)T^{-1} = -\vec{M}_e$$

$$T(\vec{j}_e)T^{-1} = -\vec{j}_e, \quad T(\Phi_m)T^{-1} = -\Phi_m, \quad T(\vec{B})T^{-1} = -\vec{B} \tag{20}$$

On applying the time reversal transformation to field associated with dyon. We get

$$T(\vec{j})T^{-1} = -\vec{j}, \quad T(\vec{V})T^{-1} = -\vec{V}, \quad T(\rho)T^{-1} = \rho$$
$$T(\vec{P})T^{-1} = \vec{P}, \quad T(\vec{M})T^{-1} = -\vec{M} \tag{21}$$

Thus, the covariant forms of Maxwell equations of dyon remain invariant under the time reversal symmetry. But Magnetization vector for dyon looses their invariance under time reversal transformation.

Symmetries of Dyon under Charge Conjugation

Charge conjugation describes the effect on a system of replacing every particle by its antiparticle. This can be thought of as reflecting it in a "mirror" that reverses the signs of all charges, classically; charge conjugation replaces positive charges by negative charges and vice-versa. Since electric and magnetic fields have their origins in charges, one can reverse these fields by applying the property of charge conjugation. The physical quantities like displacement (x), time (t), are invariant meanwhile, the physical quantities related to electric magnetic and magnetic charge change their sign under charge symmetry as,

$$C(e)C^{-1} = -e, \quad C(g)C^{-1} = -g, \quad C(\rho_e)C^{-1} = -\rho_e$$
$$C(\rho_m)C^{-1} = -\rho_m, \quad C(\rho)C^{-1} = -\rho, \quad C(\vec{j}_e)C^{-1} = -\vec{j}_e,$$
$$C(\vec{j}_m)C^{-1} = -\vec{j}_m, \quad C(\vec{j})C^{-1} = -\vec{j}, \quad C(\vec{E})C^{-1} = -\vec{E},$$
$$C(\vec{P}_m)C^{-1} = -\vec{P}_m, \quad C(\vec{P})C^{-1} = -\vec{P}, \quad C(\vec{M})C^{-1} = -M,$$
$$C(\vec{\psi})C^{-1} = -\Psi \tag{22}$$

We see that the covariant equations of dyon are invariant under C transformations. But Polarization and magnetization vector of dyon looses their invariance under C transformation.

Symmetries of Dyon under combined operator of Parity and Time Reversal (PT)

Parity (P) and time reversal (T) combine to constitute a fundamental symmetry called CP invariane. The most widely known symmetry is based on group theory is called PT symmetry. In case of PT symmetry current density, charge density are invariant under the simultaneous action of space and time reflection operators and [15, 16] i.e.

$$PT(\rho_m)PT^{-1} = \rho_m, \quad PT(\rho)PT^{-1} = \rho, \quad PT(\vec{j}_m)PT^{-1} = \vec{j}_m,$$
$$PT(\vec{j})PT^{-1} = \vec{j} \tag{23}$$

On the other hand, the following physical quantities show variable with their sign

$$PT(\vec{E})PT^{-1} = -\vec{E} \quad PT(\vec{B})PT^{-1} = -\vec{B}, \quad PT(\vec{P}_e)PT^{-1} = -\vec{P}_e,$$
$$PT(\bar{P}_m)PT^{-1} = -\vec{P}_m \quad PT(\vec{P})PT^{-1} = -\vec{P}, \quad PT(\vec{M})PT^{-1} = -\vec{M} \tag{24}$$

The combined operation of PT transformation also shows the invariance of the covariant forms of Maxwell's equations. But Polarization and magnetization change their sign under this symmetry.

Symmetries of Dyon under combined operator of charge conjugation and Parity

Associated with the conservation laws which govern the behavior of physical particles, charge conjugation (C) and parity (P) combine to constitute a fundamental symmetry called CP invariance [17-20, 21, 22]. The following quantities change their sign under the combined effect of CP i.e.

$$CP(\vec{j}_m)CP^{-1} = -\vec{j}_m \quad CP(\rho_e)CP^{-1} = -\rho_e \quad CP(\vec{P}_m)CP^{-1} = -\vec{P}_m$$
$$CP(\vec{M}_e)CP^{-1} = -\vec{M}_e \quad CP(M) = -M \tag{25}$$

The following quantities are invariant under the combined effect of CP i.e.

$$CP(\rho_m)CP^{-1} = \rho_m, \quad CP(\vec{j}_e)CP^{-1} = \vec{j}_e \quad CP(\phi_m)CP^{-1} = \phi_m,$$
$$CP(\vec{P}_e)CP^{-1} = \vec{P}_e, \quad CP(\vec{M}_m)CP^{-1} = \vec{M}_m, \quad CP(\vec{P})CP^{-1} = \vec{P} \tag{26}$$

As such, the covariant form of Maxwell equations of dyon and Polarization vector for dyon are invariant under CP symmetry. But magnetization vector of dyon looses their invariance under CP transformation.

Symmetries of Dyon under combined operator of charge conjugation and Parity

We have seen that there are three symmetries which usually, but not always, hold are those of charge conjugation (C), parity (P) and time reversal (T). The following quantities are under the combined effect of CPT i.e.

$$CPT(\vec{E})CPT^{-1} = \vec{E}, \quad CPT(\vec{B})CPT^{-1} = \vec{B},$$
$$CPT(\rho_e)CPT^{-1} = -\rho_e \quad CPT(\vec{M})CPT^{-1} = \vec{M}$$
$$CPT(\rho_m)CPT^{-1} = -\rho_m, \quad CPT(\vec{j}_e)CPT^{-1} = -\vec{j}_e,$$

$$CPT(\vec{E})CPT^{-1} = \vec{E}, \quad CPT(\vec{P}_e)CPT^{-1} = \vec{P}_e$$
$$CPT(\vec{M}_e)CPT^{-1} = \vec{M}_e, \quad CPT(\vec{P}_m)CPT^{-1} = \vec{P}_m$$
$$CPT(\vec{M}_m)CPT^{-1} = \vec{M}_m, \quad CPT(\vec{P})CPT^{-1} = \vec{P} \qquad (27)$$

The generalized field equations and covariant form of dyon is invariant under the combined operation (CPT). Polarization and magnetization vector of dyon are also invariant under CPT transformation.

Conclusion

We have observed that the Maxwell equation (1-4) are invariant under discrete symmetries like parity P, time reversal T, charge conjugation (C), parity–time reversal operation PT, charge conjugation–parity operation CP and combined operation (CPT). But when we take the generalized fields of dyon (14), we observe that the field equations are no more invariant under P symmetry, T symmetry, and combine effects of CP and PT. Polarization and magnetization vector terms of dyon are invariant under the combined operation (CPT). In other words, we may conclude that CPT invariance is an exact symmetry for generalized fields of dyon, polarization and magnetization vector of dyon. As such the statement of Ludens–Pauli–Schwinger [23] that invariance under Lorentz transformations implies CPT invariance holds good for the case of dyon.

References

1. Sozzi, M.S.: Discrete Symmetries and CP Violation. Oxford University Press, Oxford (2008) 2.
2. Roy, A.: Resonance 10, 164 (2005).
3. Leff, H.S., Rex, A.F.: Maxwell's Demon:Entropy,Information, computing. IOP, Bristol (1990).
4. Leff, H.S., Rex, A.F.: Maxwell's Demon, 2: Entropy, Classical and Quantum Information. IOP, Bristol (2003).
5. Singh, J., Bisht, P.S., Negi, O.P.S.: Commun. Phys. 17, 83 (2007).
6. Singh, J., Bisht, P.S., Negi, O.P.S.: J. Phys. A, Math.Theor. 40, 9137 (2007).
7. Singh, J., Bisht, P.S., Negi, O.P.S.: Commun. Phys. 17, 129 (2007).
8. Bisht, P.S., li Tianjun., Pushpa., Negi, O.P.S.: Discrete Symmetries and generalized fields of dyons. Int. J. Theor. Phys. 49, 1370-1383 (2010)
9. Dirac, P. A. M., Proc. Royal Society London, A133 (1931), 60.
10. Barker, W.A. and Graziani, Frank.: Am. J. Phys., 46 (1978).
11. Rajput, B. S. and Joshi, D. C.: Had. J., 4 (1981), 1805.
12. Rajput, B. S. and Negi,O.P,: Indian J. Phys., A53 (1979), 274.
13. Gunawant, R., Rajput, B.S.: IlNuovoCimento A 86, 111 (1985).
14. Bisht, P.S., Negi, O.P.S.: Proceedings of 6th International Conference on Nuclear and Particle Physics, Egyptian Nuclear Physics Association, Luxor, Egypt, Nov. 17–21, 2007, pp. 405–420 (2008).
15. Jackson, J. D.: Classical Electrodynamics,Willey,New York (1975).
16. Rosen, J.: Am. J. Phys. 41, 586 (1973).
17. Branco, G.C., Lavoura, L., Silva, J.P.: CP Violation. Clarendon, Oxford (19).
18. Bigi, I., Sanda, A.: CP Violation. CambridgeUniversity Press, Cambridge (1999).
19. Beyer, M.: CP Violation in Particle, Nuclear and Astrophysics. Springer, Berlin (2002).
20. Streater, R.F., Wightman, A.S.: PCT, Spin Statistics and All That. Benjamin/Cummings, Redwood City (1964).
21. Bender, C.M., Boetter, S.: Phys. Rev. Lett. 24, 5243 (1998).
22. Bender, C.M., Boetter, S.: J. Phys. A, Math. Gen. 31, L273 (1998).
23. Ludens, G., Pauli, W., Schwinger, J.http://hyperphysics.phyastr.gsu.edu/hbase/Particles/cpt.html.