Protecting entanglement by using single qubit weak measurement and quantum measurement reversal against one and two sided amplitude damping.

Balwant Singh Airy[1] and U.C.Johri[2]
[1,2]Department of Physics, G. B. P.U.A.T.
Pantnagar, India
Email - balwantsingh.airy46@gmail.com

Abstract - In this paper we have studied the effect of one and two sided amplitude damping on the entanglement of Bell's like states by measuring their concurrence and compare them .We found that in the case of two sided amplitude damping, state $\alpha|01\rangle+\beta|10\rangle$ shows more protection of entanglement compare to the state $\alpha|00\rangle+\beta|11\rangle$. While in the case of one sided amplitude damping both state perform equally. We have also shown that entanglement can be protected by using one qubit weak measurement and Quantum measurement reversal in both cases . We found that in the case of two sided amplitude damping after applying one qubit weak measurement and Quantum measurement reversal ,state $\alpha|01\rangle+\beta|10\rangle$ is protected more compare to the state $\alpha|00\rangle+\beta|11\rangle$ and similar results are obtained in the case of one sided amplitude damping.

Key Words: *Decoherence, Amplitude damping, Entanglement, concurrence, Weak measurement, Quantum measurement reversal*

1 Introduction

Entanglement is the vital for quantum computing, quantum Cryptography quantum teleportation[1-3],quantum secret sharing[4-6],telecloning[7-9] etc. But quantum entanglement can be easily damaged by decoherence, which occur due to the interaction of quantum system with the surrounding [8]. There are many channel which are responsible for decoherence, eg- amplitude damping, phase damping, depolarization channel etc[9,10] . In this paper we have used two sided and one sided amplitude damping. It modeled the dissipative interaction of qubits with its zero temperature environment. It is described by following quantum map

$$|0\rangle_S|0\rangle_E \rightarrow |0\rangle_S|0\rangle_E$$

$$|0\rangle_S|0\rangle_E \rightarrow D^{1/2}|1\rangle_S|0\rangle_E+(1-D)^{1/2}|0\rangle_S|1\rangle_E$$

The action of the above map can be described by the set of operator known as Kraus operator, which is given by

$$E_0=\begin{bmatrix}1 & 0\\ 0 & \sqrt{1-D}\end{bmatrix}\quad E_1=\begin{bmatrix}0 & \sqrt{D}\\ 0 & 0\end{bmatrix}$$

Where D is decoherence strength parameter and it varies from 0 to 1.

There are number of way of protecting entanglement eg: quantum error correction[11],Decoherence free sub space[12], dynamical decoupling[13] etc. In this paper we are using one qubit weak measurement and quantum measurement reversal[14].This paper is organized as follows. In section 2 we have applied the one and two sided amplitude damping to state $I\psi\rangle = \alpha|00\rangle+\beta|11\rangle$ and measure its concurrence. then we apply the single qubit weak measurement and Quantum measurement reversal in both cases. In section 3 we repeat the same process with state $I\varphi\rangle= \alpha|01\rangle+\beta|10\rangle$ and in section 4 we discuss the result and in section 5 we concluded the paper.

2 for state $|\psi\rangle = \alpha|00\rangle+\beta|11\rangle$ (STATE 1)

Let consider the two qubit quantum state in maximal entangle state ,which is given by

$$|\psi\rangle=\alpha|00\rangle+\beta|11\rangle \tag{1}$$

Its density matrix can be written as

$$\rho=\begin{bmatrix}\alpha^2 & 0 & 0 & \alpha\beta\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ \beta\alpha & 0 & 0 & \beta^2\end{bmatrix} \tag{2}$$

Where $\alpha^2+\beta^2=1$

The state after applying decoherence is given by

$$\varepsilon_{AD}(\rho)=A_0\rho A_0^*+A_1\rho A_1^*+A_2\rho A_2^*+A_3\rho A_3^* \tag{3}$$

Where A_0, A_1, A_2, A_3 are Kraus operator for two sided amplitude damping noise and written as

$$A_0=E_0\otimes E_0=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \sqrt{1-D} & 0 & 0\\ 0 & 0 & \sqrt{1-D} & 0\\ 0 & 0 & 0 & 1-D\end{bmatrix},$$

$$A_1=E_0\otimes E_1=\begin{bmatrix}0 & \sqrt{D} & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \sqrt{D(1-D)}\\ 0 & 0 & 0 & 0\end{bmatrix}$$

$$A_2 = E_1 \otimes E_0 = \begin{bmatrix} 0 & 0 & \sqrt{D} & 0 \\ 0 & 0 & 0 & \sqrt{D(1-D)} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_3 = E_1 \otimes E_1 = \begin{bmatrix} 0 & 0 & 0 & D \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (4)$$

For simplicity we have assumed that decoherence strength are same in both side ie D_1=D_2 =D

Now the resultant density matrix is given by

$$\varepsilon_{AD}(\rho) = \begin{bmatrix} \alpha^2+\beta^2 D^2 & 0 & 0 & \alpha\beta(1-D) \\ 0 & \beta^2 D(1-D) & 0 & 0 \\ 0 & 0 & \beta^2 D(1-D) & 0 \\ \beta\alpha(1-D) & 0 & 0 & \beta^2(1-D)^2 \end{bmatrix}$$

Now entanglement can be measure by measuring the concurrence, which is calculated be

$$C = 2(1-D)\beta(\alpha - D\beta) \quad (5)$$

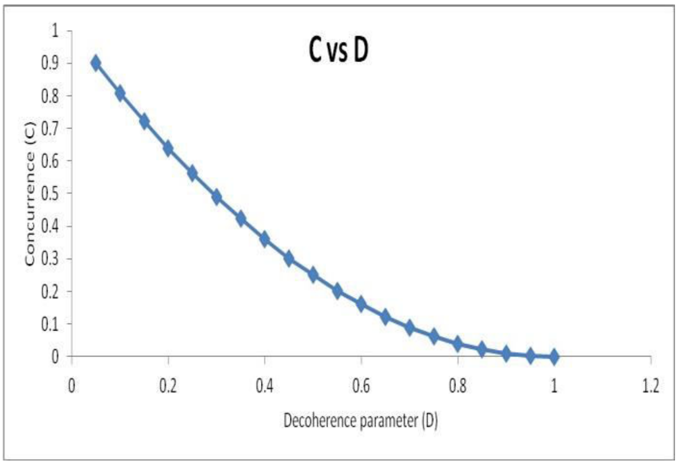

Figure 1 C Vs D

Let us now protect the entanglement by applying one qubit weak measurement and Quantum measurement reversal. we apply weak measurement before system undergoes amplitude damping decoherence.

The two qubit weak measurement operator can be written as

$$M_W = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-M} \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$M_W = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \sqrt{1-M} & 0 \\ 0 & 0 & 0 & \sqrt{1-M} \end{bmatrix}$$

Now after applying amplitude decoherence we apply Quantum measurement reversal .The two qubit reversing measurement operator is given by

$$M_{rev} = \begin{bmatrix} \sqrt{1-M_r} & 0 \\ 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$M_{rev} = \begin{bmatrix} \sqrt{1-Mr} & 0 & 0 & 0 \\ 0 & \sqrt{1-M_r} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where M_r=M-(1-M)D.

Assuming that reversing measurement is optimal, the two qubit state after the sequence of weak measurement, amplitude damping and Quantum measurement reversal the two qubit state is given by

$$\varepsilon_{AD}(\rho) = \frac{1}{A}\begin{bmatrix} \alpha^2+\beta^2 D^2(1-M)^2 & 0 & 0 & \alpha\beta\sqrt{1-D} \\ 0 & \beta^2 D(1-M)(1-D) & 0 & 0 \\ 0 & 0 & \beta^2 D & 0 \\ \beta\alpha\sqrt{1-D} & 0 & 0 & \beta^2(1-D) \end{bmatrix}$$

Where A=$1+\beta^2 D^2(1-M)+\beta^2 D(1-D)(1-M)$

Now again entanglement can be measure by measuring the concurrence, which is calculated be

$$C = \frac{2\left[\alpha\beta\sqrt{1-D} - \beta^2 D^2\sqrt{(1-D)(1-M)}\right]}{\left[1+\beta^2 D^2(1-M) - \beta^2 D(1-D)(1-M)\right]} \quad (6)$$

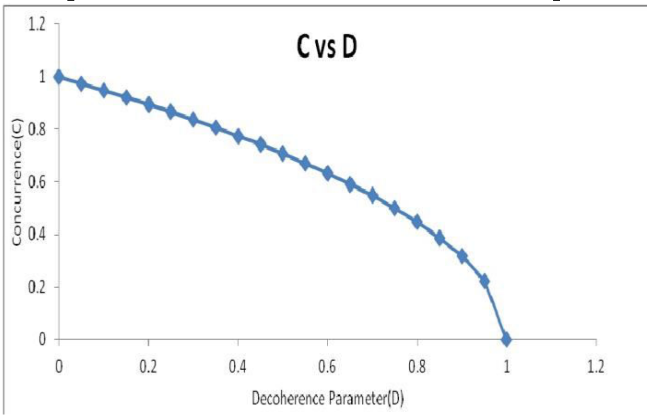

Figure 2 C Vs M (D=0.5)

Now for one sided amplitude damping , The state after applying decoherence is given by

$$\varepsilon_{AD}\left(\rho'\right) = A_0 \rho A_0^* + A_1 \rho A_1^*$$

Where A_0, A_1 are Kraus operator for one sided amplitude damping noise and written as

$$A_0 = E_0 \otimes I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \sqrt{1-D} & 0 \\ 0 & 0 & 0 & 1-D \end{bmatrix},$$

$$A_1 = E_1 \otimes I = \begin{bmatrix} 0 & 0 & \sqrt{D} & 0 \\ 0 & 0 & 0 & \sqrt{D} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad (5)$$

Now the resultant density matrix is given by

$$\varepsilon_{AD}(\rho) = \begin{bmatrix} \alpha^2 & 0 & 0 & \alpha\beta\sqrt{1-D} \\ 0 & \beta^2 D & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \beta\alpha\sqrt{1-D} & 0 & 0 & \beta^2(1-D)^2 \end{bmatrix}$$

Now entanglement can be measure by measuring the concurrence, which is calculated be

$$C = 2\alpha\beta\sqrt{1-D} \qquad (7)$$

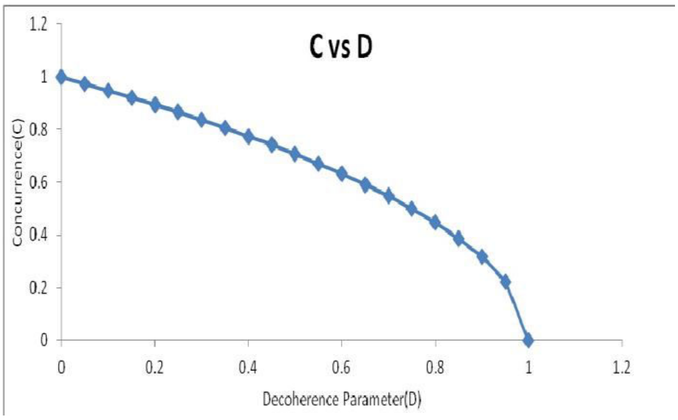

Figure 3 C Vs D

Repeating the all process as in the above , the resultant density matrix is given by

$$\varepsilon_{AD}(\rho) = \frac{1}{A} \begin{bmatrix} \alpha^2 & 0 & 0 & \alpha\beta \\ 0 & \beta^2(1-M) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \beta\alpha & 0 & 0 & \beta^2 \end{bmatrix}$$

Where $A = 1 + \beta^2(1-M)$

Now entanglement can be measure by measuring the concurrence, which is calculated be

$$C = \frac{2\alpha\beta}{1+\beta^2(1-M))} \qquad (8)$$

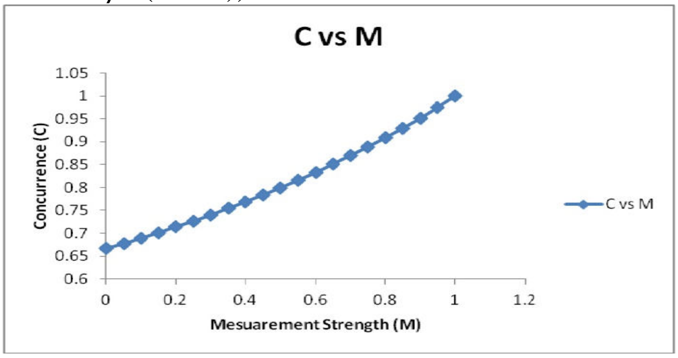

Figure 4 C Vs M (D=0.5)

3 for state |φ⟩ =α|01⟩+β|10⟩ (STATE 2)

Let consider the two qubit quantum state which is given by

|φ⟩ = α|01⟩+β|10⟩

Its density matrix can be written as

$$\rho = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \alpha^2 & \alpha\beta & 0 \\ 0 & \beta\alpha & \beta^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The state after applying decoherence is given by

$$\varepsilon_{AD}\left(\rho\right) = A_0 \rho A_0^* + A_1 \rho A_1^* + A_2 \rho A_2^* + A_3 \rho A_3^*$$

(3)

Where A_0, A_1, A_2, A_3 are Kraus operator for two sided amplitude damping noise

$$\varepsilon_{AD}(\rho) = \begin{bmatrix} \alpha^2 D + \beta^2 D & 0 & 0 & 0 \\ 0 & \alpha^2(1-D) & \alpha\beta(1-D) & 0 \\ 0 & \beta\alpha(1-D) & \beta^2(1-D) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Where $\alpha^2 + \beta^2 = 1$

Now entanglement can be measure by measuring the concurrence, which is calculated be

$$C = 2\alpha\beta(1-D) \qquad (9)$$

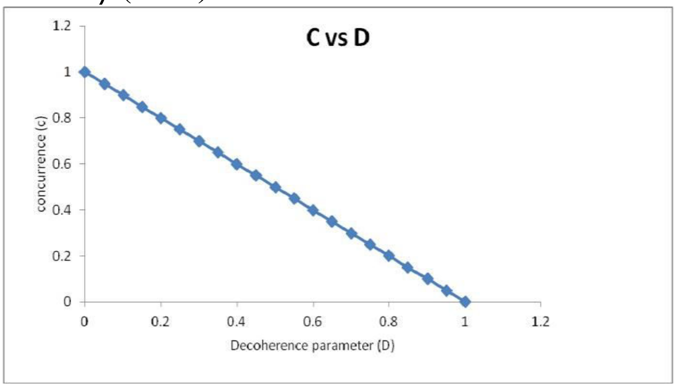

Figure 5 C Vs D

Now as earlier we apply sequence of weak measurement, amplitude damping and Quantum measurement reversal and the two qubit state is given by

$$\varepsilon_{AD}(\rho)=\frac{1}{A'}\begin{bmatrix} \left[\alpha^2 D+\beta^2 D(1-M)\right] & 0 & 0 & 0 \\ 0 & \alpha^2(1-D) & \alpha\beta\sqrt{1-D} & 0 \\ 0 & \beta\alpha\sqrt{1-D} & \beta^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Where $A'=1+\beta^2 D(1-M)$

Now entanglement can be measure by measuring the concurrence, which is calculated be

$$C=\frac{2\alpha\beta\sqrt{1-D}}{1+\beta^2 D(1-M)} \tag{10}$$

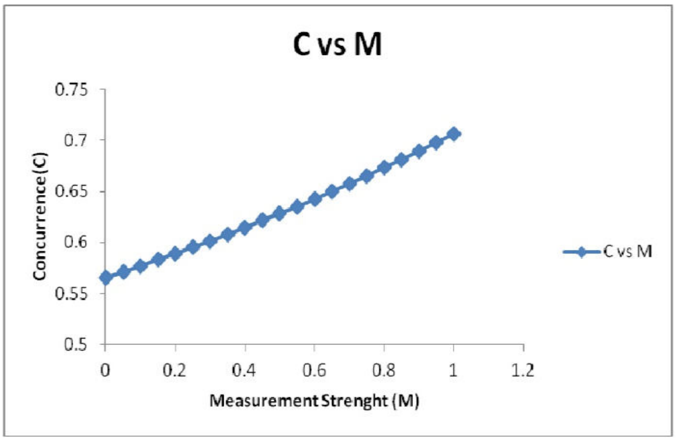

Figure 6 C Vs M (D=0.5)

Now for one sided amplitude damping , repeating all the process as earlier the resultant density matrix is given by

$$\varepsilon_{AD}(\rho)=\begin{bmatrix} \beta^2 D & 0 & 0 & 0 \\ 0 & \alpha^2 & \alpha\beta\sqrt{1-D} & 0 \\ 0 & \beta\alpha\sqrt{1-D} & \beta^2(1-D) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Now entanglement can be measure by measuring the concurrence, which is calculated be

$$C=2\alpha\beta\sqrt{1-D} \tag{11}$$

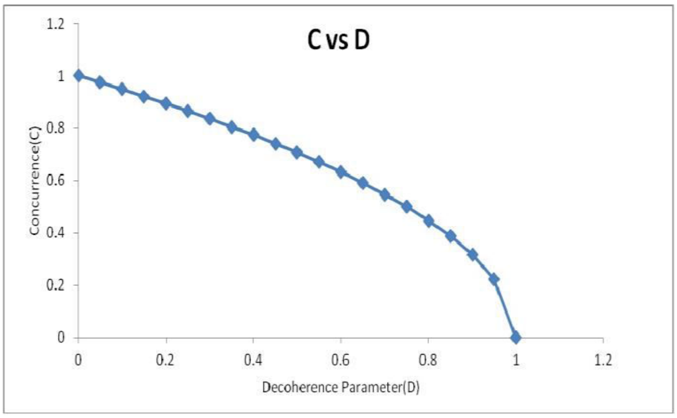

Figure 7 C Vs D

Now as earlier we apply sequence of weak measurement, amplitude damping and Quantum measurement reversal and the two qubit state is given by

$$\varepsilon_{AD}(\rho)=\frac{1}{A'}\begin{bmatrix} \beta^2 D(1-M) & 0 & 0 & 0 \\ 0 & \alpha^2 & \alpha\beta & 0 \\ 0 & \beta\alpha & \beta^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Where $A'=1+\beta^2 D(1-M)$

Now entanglement can be measure by measuring the concurrence, which is calculated be

$$C=\frac{2\alpha\beta}{1+\beta^2 D(1-M)} \tag{12}$$

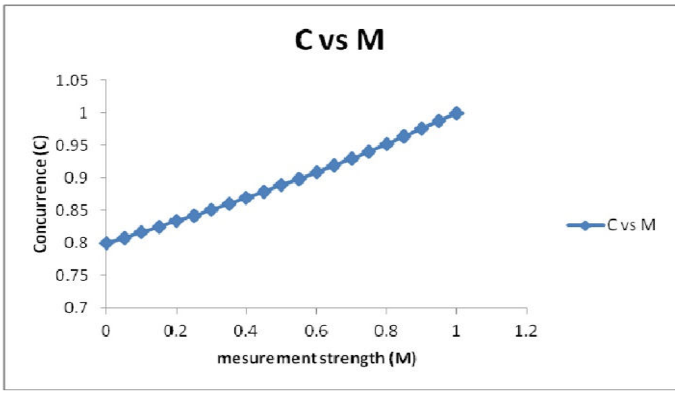

Figure 8 C Vs M (D=0.5)

4 DISCUSSION

First we explain the results obtained from two sided amplitude damping. from figure 1 and figure 5 we find that on applying same decoherence on state $\alpha|00\rangle+\beta|11\rangle$ and $\alpha|01\rangle+\beta|10\rangle$ may contain unequal entanglement their comparison is given in figure below.

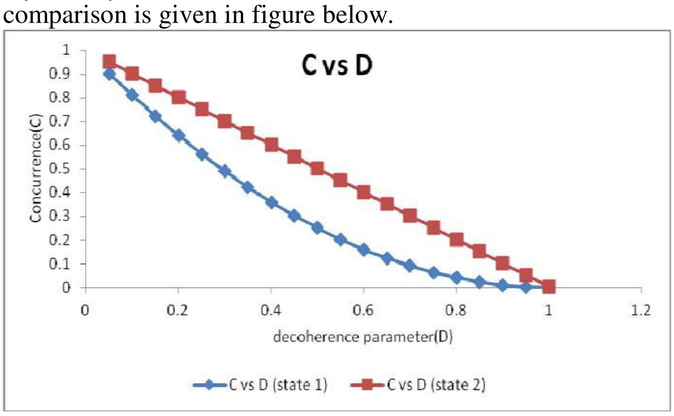

Figure 9 C Vs D

It is clear that state $\alpha|01\rangle+\beta|10\rangle$ shows more protection of entanglement compare to the state $\alpha|00\rangle+\beta|11\rangle$ to amplitude damping .

As from figure 2 and figure 6 we find that same decoherence and entanglement protection protocol of two LO equivalent state $\alpha|00\rangle+\beta|11\rangle$ and $\alpha|01\rangle+\beta|10\rangle$ may contain unequal entanglement their comparison is given in figure below.

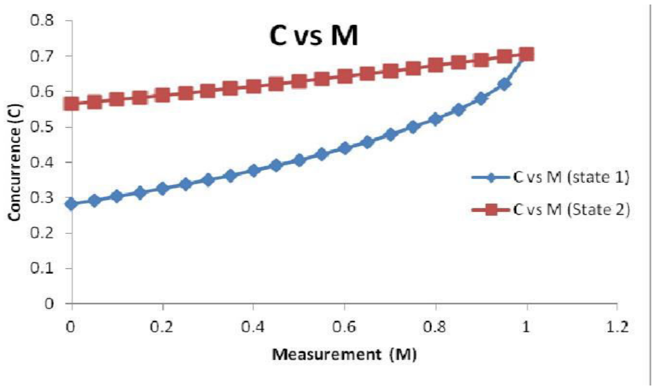

Figure 10 C Vs M (D=0.5)

It is clear that state $\alpha|01\rangle+\beta|10\rangle$ shows more protection of entanglement compare to the state $\alpha|00\rangle+\beta|11\rangle$ to two sided amplitude damping .

Now we explain the results obtained from one sided amplitude damping. from figure 3 and figure 7 it is clear that state $\alpha|01\rangle+\beta|10\rangle$and $\alpha|00\rangle+\beta|11\rangle$ shows exactly same level of protection of entanglement to one sided amplitude damping as shown in figure .

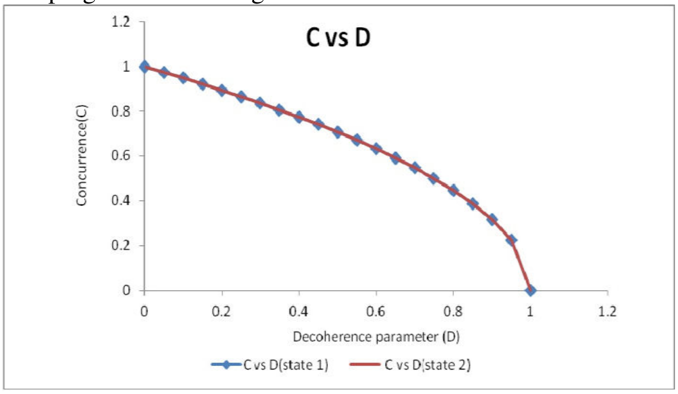

Figure 11 C Vs D

As from figure 4 and figure 8 we find that same decoherence and entanglement protection protocol of two LO equivalent state $\alpha|00\rangle+\beta|11\rangle$ and $\alpha|01\rangle+\beta|10\rangle$ may contain unequal entanglement their comparison is given in figure below.

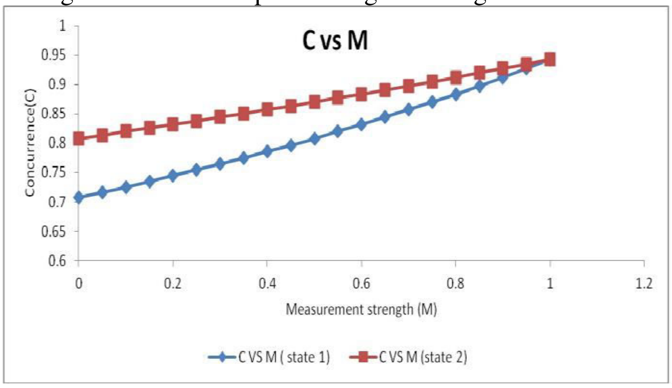

Figure 12 C Vs M (D=0.5)

It is clear that state $\alpha|01\rangle+\beta|10\rangle$ shows more protection of entanglement compare to the state $\alpha|00\rangle+\beta|11\rangle$ to one sided amplitude damping. Further from figure 9,10,11,12 it can be seen that state $\alpha|00\rangle+\beta|11\rangle$ in the case of two sided amplitude damping are showing maximum entanglement, when it is not initially maximally entangled which show that non maximally entangled state can be used for better entanglement distribution [15-20]

5 conclusion

We find that In the case of two sided amplitude damping channel state $\alpha|01\rangle+|10\rangle$ protect entanglement better than $\alpha|00\rangle+\beta|00\rangle$ before and after applying the weak measurement and quantum measurement reversal. while in the case of one sided amplitude damping before applying the weak measurement and quantum measurement reversal both state protected same amount of entanglement while after applying them $\alpha|01\rangle+|10\rangle$ behave better than first state.

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Balwant Singh Airy, I am pursuing my Ph.D From G.B.P.U.A.T, Pantnagar, Uttarakhand under the supervision of Dr. U. C. Johri in Quantum Computation and communication.

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